



MACHAKOS UNIVERSITY

University Examinations for 2017/2018

SCHOOL OF BUSINESS AND ECONOMICS

DEPARTMENT OF ECONOMICS

SECOND YEAR FIRST SEMESTER EXAMINATIONS FOR

BACHELOR OF ECONOMICS AND FINANCE

BACHELOR OF ECONOMICS AND STATISTICS

BACHELOR OF ECONOMIS

EES 200: MATHEMATICS FOR ECONOMICS II

DATE: 13/12/2017

TIME: 8:30 – 10: 30 AM

INSTRUCTIONS: Answer Question ONE and any other TWO questions

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) Give the following national income identity for open economy: (5 marks)

$$Y = C + I_0 + G_0 + X_0 - M$$

$$C = c_0 + c_1(Y^d)$$

$$T = t_0 + t_1Y$$

$$M = m_0 + m_1Y$$

$$Y^d = Y - T$$

Y , C and T are endogenous variables while I_0 and G_0 are exogenous variables.

Find equilibrium Y , C and T

- b) Given the following demand and supply and equilibrium functions (5 marks)

$$Q_d = a - bP$$

$$Q_s = -c + dP$$

$$Q_d = Q_s$$

Find equilibrium p and q and evaluate how the two changes with parameter

- c) Find the derivatives of the following problems.

i. $y = \ln x$ (2 marks)

ii. $y = \ln(ax)$ (2 marks)

iii. $y = \ln x^2$ (2 marks)

iv. $Y = \log_a X$ (2 marks)

v. $Y = \log_5(2X^3 + 3X)$ (2 marks)

- d) Partial derivatives of utility function yields marginal utility whereby second derivative determines the nature of Marginal utility. Explain how marginal utility can be positive and to some level negative, use graphs to support your points. (5 marks)
- e) What does return to scale mean? And define the three types of return to scale. (5 marks)

QUESTION TWO (20 MARKS)

- a) A system of two simultaneous equations is given as: (5 marks)

$$X + 2Y = 10$$

$$5X + 8Y = 40$$

Determine the values of X and Y using the Cramer's rule

b) Find the total differential of the following utility functions

i) $Y = 4X_1 + 10X_2 X_1^2 + 3X_2^3$ (2 marks)

ii) $Z = 5XY + 2X^2Y - 8Y^2X$ (2 marks)

iii) $q = \frac{x^\alpha l^\beta}{x^\phi}$ (2 marks)

c) Assume that a demand function is given as: (5 marks)

$$P = 40 - 2Q$$

Determine the consumer surplus when **market price = 10**

d) Find the integral of the following:

i. $\int Ax^n . dx$ (1 mark)

ii. $\int e^x . dx$ (1 mark)

iii. $\int \frac{1}{x} dx$ (1 mark)

iv. $\int dx$ (1 mark)

QUESTION THREE (20 MARKS)

a) A production function is given as: (8 marks)

$$Q = 80K^{0.2}L^{0.5}$$

Determine the marginal product of labour, marginal product of capital, the slope of the production function and whether the marginal products are increasing or decreasing.

b) What is the Homogeneity of a function? (2 marks)

c) Determine the homogeneity of the function given as: (3 marks)

$$Y = X^2 + 5XW + W^2$$

- d) A Simple national income model is given as: (7 marks)

$$Y = C + I_0 + G_0$$

$$C = c_0 + c_1(Y^d)$$

$$Y^d = Y - T$$

$$C = c_0 + c_1(Y - T)$$

$$T = t_0 + t_1 Y$$

$Y = \text{national income}, C = \text{consumption expenditure},$

$Y^d = \text{disposable income}, T = \text{tax}$

where $c_0 > 0$ $0 < c_1 < 1$

$t_0 > 0$ $0 < t_1 < 1$

Use Cramer's rule to determine the expressions for the equilibrium values of the endogenous variables Y, C and T .

QUESTION FOUR (20 MARKS)

- a) Determine the nature of returns to scale for the following function: (6 marks)

$$Q = 80K^{0.2}L^{0.5}$$

$$Q = A L^{\frac{1}{2}} K^{\frac{3}{4}}$$

- b) A demand function is given as: (9 marks)

$$Q_A = 90 - 2P_A + 0.4P_B + 0.1Y$$

Find the price, income and cross price elasticities of demand when:

$$P_A = 6 \quad P_B = 10 \quad Y = 1000$$

- c) Explain Euler's theorem (2 marks)

- d) Given the production function below proof the Euler's theorem algebraically. (3 marks)

i) $Q = AK^\alpha L^\beta$ (1 mark)

ii) $Q = 3K^3 - 5KL^2 - L^3$ (2 marks)

QUESTION FIVE (20 MARKS)

- a) Assume the following equation is a maximization problem. (6 marks)

$$Z = x^2 + y^2 + xy$$

Subject to : $x + 3y = 13$

Require : Solve for x, y and λ

- b) Given a utility function $U=5xy$, and a budget constraint given as $5x+y=30$, determine the levels of x and y that will maximize the utility of the consumer. (4 marks)

- c) The production function for a firm is given as $Q = 12L^{0.5}K^{0.5}$. If the firm wants to produce an output of 240 units, find the optimal values of labour and capital that will minimize the total cost of production given that labour cost per unit is 25 dollars and capital cost per unit is 50 dollars. (6 marks)

- d) Find determinants of the following: (4 marks)

i. $\begin{bmatrix} 8 & 0 \\ 4 & 1 \end{bmatrix}$ (1 mark)

ii. $\begin{bmatrix} 4 & 2 & 0 \\ 8 & 9 & 2 \\ 1 & 5 & 3 \end{bmatrix}$ (2 marks)