MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCENCE

THIRD YEARSPECIAL/SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE
BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING
BACHELOR OF SCIENCE IN MATHEMATICS
BACHELOR OF ECONOMICS AND STATISTICS
BACHELOR OF EDUCATION
BACHELOR OF ARTS

SMA 336: ORDINARY DIFFERENTIAL EQUATION II

DATE: 27/9/2019 **TIME: 11:00 – 1:00 PM**

Instructions to the Candidate:

Attempt question ***one (compulsory)*** and any other ***two questions***.

Question 1 (30mks)

a.) i.) define the Wronskian of a set of functions $F = \{f_1\ ,\ f_2\ ,\ f_3\ ,\ \ldots\ ,\ f_n\}$(3 marks)

ii.) Show that the Wronskian of the functions x^2 and $x^2 \ln x$ is non-zero.(4 marks)

b.) State the existence and uniqueness theorem for the nth order linear differential equation. (2 marks)

c.) Consider the second order linear differential equation $(2x+1)\frac{d^2 y}{dx^2} - 4(x+1)\frac{dy}{dx} + 4y = 0$.

Determine the linearly independent solution of the equation by reducing the order. (5 marks)

d.) Consider the differential equation $3x^2 dx + 3y^2 dy - \left(x^3 + y^3 + e^{2z}\right)dz = 0$; solve the equation by taking one of the variables as a constant. (5 marks) e.) i.) consider the nth order differential equation

$a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + ... + a_{n-1} \frac{dy}{dx} + a_n y = F$. State three conditions that the equation could be an nth order linear differential equation. (3 marks)

ii.) determine a first integral of the differential equation $\frac{dy}{dx}\frac{d^2 y}{dx^2} - x^2 y \frac{dy}{dx} = xy^2$

(3 marks)

f.) solve the equation $y'' - y = x$ at $x = 0$ using the power series solution method.

(3 marks)

Question 2(20mks)

a.) define linearly independent solution of a linear system of differential equations.(3 marks)

b.) consider the system of differential equations

$$\frac{dx}{dt} = 2x - y$$
$$\frac{dy}{dt} = 3x + 6y$$

i.) show that

$$\begin{array}{l} x = e^{5t} \\ y = -3e^{5t} \end{array} \quad \text{and} \quad \begin{array}{l} x = e^{3t} \\ y = -e^{3t} \end{array}$$ are linearly independent solutions of the system (6 marks)

ii.) determine the solution $x = f(t)$, $y = g(t)$ of the system which is such that $f(0) = 0$, $g(0) = 2$ (4 marks)

c.) use the operator method to determine the general solution of the linear systems.

$$(D-1)x + Dy = 2t + 1$$
$$(2D+1)x + 2Dy = t$$

10 marks)

Question 3(20mks)

Consider the system of differential equations below;

$$\frac{dx}{dt} = -2x + 2x^2$$
$$\frac{dy}{dt} = -3x + y + 3x^2$$

a.) determine;

i.) the critical points (4 marks)

ii.) the nature of the critical points (3 marks) b.) linearize the system of the differential equation (8 marks)

c.) solve the system for x and y. (5 marks)

question 4 (20mks)

a.) consider the differential equation

$$x^2 \sin x \frac{d^3 y}{dx^3} - \left(3x^2 \sin x + x^3 \cos x\right)\frac{d^2 y}{dx^2} + \left(6x \sin x + 2x^2 \cos x\right)\frac{dy}{dx} - \left(6 \sin x + 2x \cos x\right)y = 0$$

i.) show that $y = x$ is a solution of the differential equation above. (3 marks)

ii.) reduce the third order equation to a second order equation and solve it.(7 marks)

b.) consider the following differential equation;

$$\left(z + z^3\right)\cos x dx - \left(z + z^3\right)dy + \left(1 + z^2\right)\left(y - \sin x\right)dz = 0$$

i.) verify the integrability condition for the equation (3 marks)

ii.) solve the equation (7 marks)

Question 5(20mks)

a.) Consider the Legendre equation $\left(1 - x^2\right)y'' - 2xy' + p\left(p+1\right)y = 0$. This equation has two independent series solution ;

$$y = c_0\left[1 - \frac{p(p+1)}{2!}x^2 + \frac{p(p-2)(p+1)(p+3)}{4!}x^4 + \ldots\right]$$
$$+ c_1\left[x - \frac{(p-1)(p+2)}{3!}x^3 + \frac{(p-1)(p-3)(p+2)(p+4)}{5!}x^5 + \ldots\right]$$

Which is denoted by $y = c_0 y_1 + c_1 y_2$. Show that the Legendre polynomial of order three is given by $p_3(x) = \frac{5}{2}x^3 - \frac{3}{2}x$ (3 marks) b.) The Rodriquez formula for Legendre's polynomial of order n is given by;

$$p_n(x) = \frac{1}{2^n n!} \frac{d}{dx^n} \left(x^2 - 1\right)^n$$

Determine the Legendre's polynomial for;

i.) $p_1(x)$ (2 marks)

ii.) $p_2(x)$ (3 marks)

c.) Prove that;

i.) $J_{-p}(x) = (-1)^p J_p(x)$ (4 marks)

ii.) Bessel's function of order zero is given by

$$J_0(x) = 1 - \frac{x^2}{2^2} + \frac{x^4}{2^2 \cdot 4^2} - \frac{x^6}{2^2 \cdot 4^2 \cdot 6^2} + \ldots$$ (4 marks)

d.) Show that the differential equation $\dfrac{d^2 y}{dx^2} + 3x\dfrac{dy}{dx} + x^3 y = 2e^x$

$$y(1) = 2 \quad , \quad y'(1) = 5$$

has a unique solution (4 marks)