MACHAKOS UNIVERSITY

University Examinations for 2021/2022 Academic Year

SCHOOL OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF MECHANICAL AND MANUFACTURING ENGINEERING

FOURTH YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)

EMM 402: MECHANICS OF MACHINES III

DATE: 7/3/2023 **TIME: 11:00-1:00 PM**

INSTRUCTIONS

This paper contains Five (5) question

Answer question ONE (compulsory) and any other two questions.

Use **CLEAR, NEAT** and **WELL-LABELLED** sketches.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Using a neat sketch, briefly explain the following terminologies:

i. Pitch circle (1 mark)

ii. Pitch circle diameter (1 mark)

iii. Pitch point (1 mark)

iv. Pitch surface (1 mark)

v. Pressure angle (1 mark)

b) State **THREE** advantages of involute gear profile and **THREE** advantages of cycloidal gear profile. (3 marks)

c) Consider a pinion driving a wheel as shown in **Fig. Q1 (c).** When the pinion rotates in clockwise direction, the contact between a pair of involute teeth begins at *K* and ends at *L*. *MN* is the common normal at the point of contacts and the common tangent to the base circles. The point *K* is the intersection of the addendum circle of wheel and the common tangent. The point *L* is the intersection of the addendum circle of pinion and common tangent.

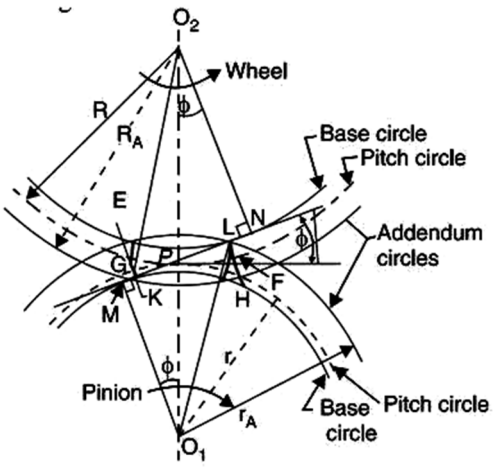

Fig. Q1 (c)

Show that length of path of contact is;

$$\sqrt{(R_A)^2 - R^2\cos^2\phi} + \sqrt{(r_A)^2 - r^2\cos^2\phi} - (R + r)\sin\phi$$

(5 marks)

and length of arc of contact is;

$$\frac{\text{Length of path of contact}}{\cos\phi}$$

(2 marks)

d) A pinion having 30 teeth drives a gear having 80 teeth. The profile of the gears is involute with 20° pressure angle, 12 mm module and 10 mm addendum. Find the length of path of contact, arc of contact and the contact ratio. (5 marks)

e) State and briefly explain four different types of gear trains depending upon the arrangement of the wheels. (4 marks)

f) The speed ratio of the gear train, as shown in **Fig. Q1 (f),** is to be 12. The module pitch of gears A and B is 3.125 mm and of gears C and D is 2.5 mm. Calculate the suitable numbers of teeth for the gears. No gear is to have less than 24 teeth. (6 marks)

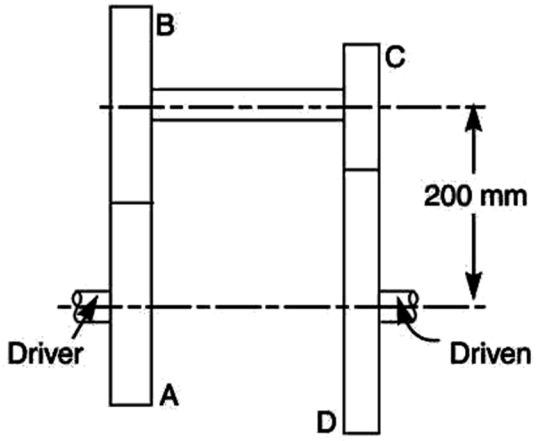

Fig. Q1 (f)

QUESTION TWO (20 MARKS)

a) In order to avoid interference, the addendum circles for two mating gears must cut the common tangent to the base circles between the points of tangency as shown in **Fig. Q2 (a)**. The limiting condition reaches, when the addendum circles of pinion and wheel pass through points *N* and *M*. Letting t = Number of teeth on the pinion, T = Number of teeth on the wheel, r = Pitch circle radius of pinion = $m \cdot t / 2$, G = Gear ratio = $T / t = R / r$ and $\emptyset$ = Pressure angle or angle of obliquity and m = Module of the teeth.

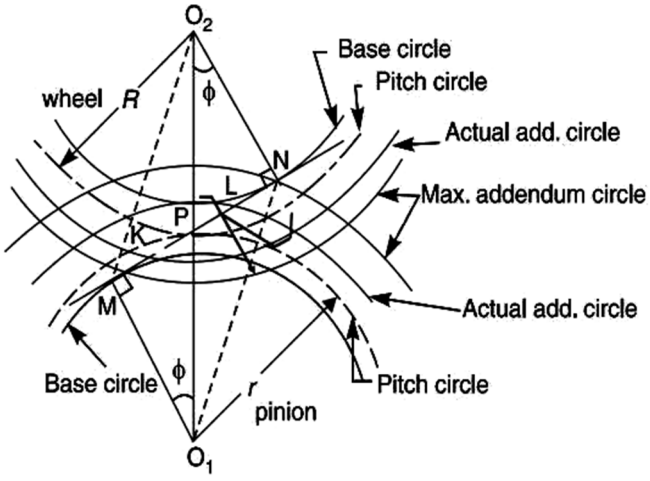

Fig. Q2 (a): Interference in involute gears

Show that minimum number of teeth t, on the pinion in order to avoid interference (for G =1), is given by;

$$t = \frac{2A_p}{\sqrt{1+3\sin^2\phi - 1}}$$

(6 marks)

b) Determine the minimum number of teeth required on a pinion, in order to avoid interference which is to gear with, (The pressure angle is 20° and a standard addendum of 1 module for the wheel)

i. A wheel to give a gear ratio of 3 to 1. (2 marks)

ii. An equal wheel. (2 marks)

c) A pair of involute spur gears with 16° pressure angle and pitch of module 6 mm is in mesh. The number of teeth on pinion is 16 and its rotational speed is 240 r.p.m. When the gear ratio is 1.75, find

i. The addenda on pinion and gear wheel

ii. The length of path of contact

iii. The maximum velocity of sliding of teeth on either side of the pitch point.

In order that the interference is just avoided. (10 marks)

QUESTION THREE (20 MARKS)

a) **Fig. Q3 (a)** shows a rack and pinion in mesh. Letting t = Minimum number of teeth on the pinion to avoid interference, r = Pitch circle radius of the pinion = $m \cdot t$ / 2, and ϕ = Pressure angle or angle of obliquity, and $A_R \cdot m$ = Addendum for rack, where is the fraction by which the standard addendum of one module for the rack is to be multiplied. Show that

(2 marks)

$$t = \frac{2 A_R}{\sin^2 \phi}$$

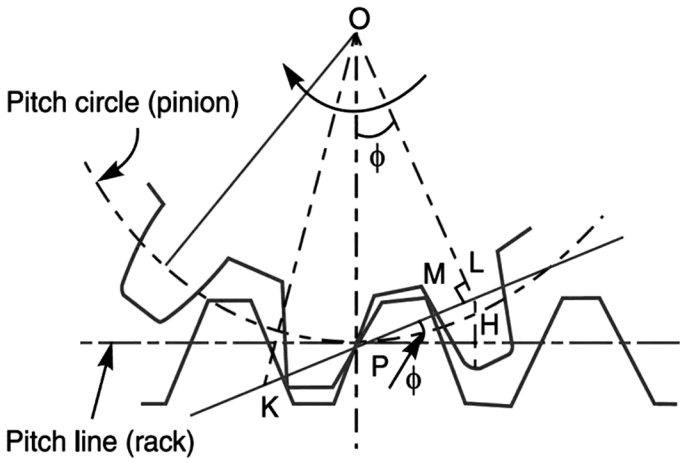

Fig. Q3 (a)

b) A pinion of 20 involute teeth and 125 mm pitch circle diameter drives a rack. The addendum of both pinion and rack is 6.25 mm.

i. Find the least pressure angle which can be used to avoid interference. (2 marks)

ii. With this pressure angle, find:

I. The length of the arc of contact. (2 marks)

II. The minimum number of teeth in contact at a time. (2 marks)

c) A pair of spiral gears 1 and 2 in mesh is shown in **Fig. Q3 (c)** below.

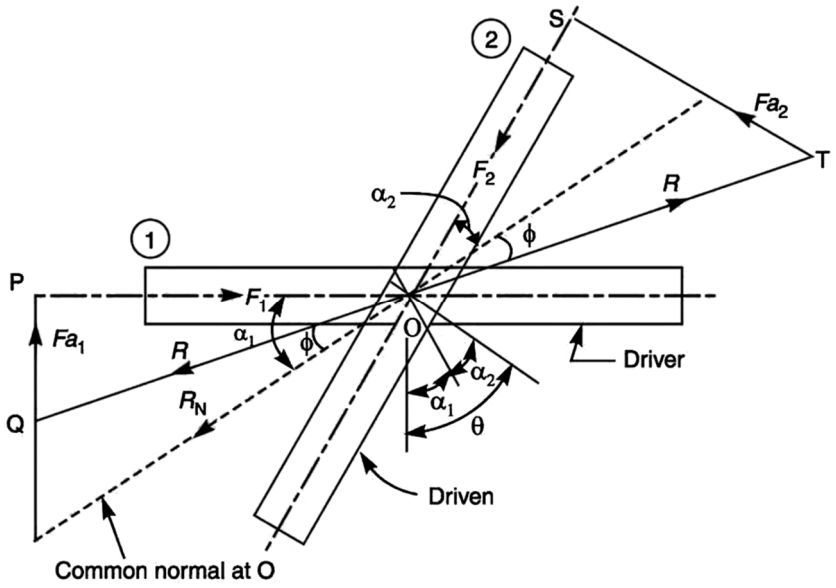

Fig. Q3 (c)

Let the gear 1 be the driver and the gear 2 the driven. The forces acting on each of a pair of teeth in contact are shown in **Fig. Q3 (c).** The forces are assumed to act at the centre of the width of each teeth and in the plane tangential to the pitch cylinders. Letting F_1 = Force applied tangentially on the driver, F_2 = Resisting force acting tangentially on the driven, F_{a1} = Axial or end thrust on the driver, F_{a2} = Axial or end thrust on the driven, R_N = Normal reaction at the point of contact, ∅= Angle of friction, R = Resultant reaction at the point of contact, θ = Shaft angle = $\alpha_1 + \alpha_2$ and η_{max} is the maximum efficiency. Show that;

$$\eta_{max} = \frac{\cos(\theta + \phi) + 1}{\cos(\theta - \phi) + 1}$$

(6 marks)

d) A pair of spiral gears is required to connect two shafts 175 mm apart, the shaft angle being 70°. The velocity ratio is to be 1.5 to 1, the faster wheel having 80 teeth and a pitch circle diameter of 100 mm.

i. Find the spiral angles for each wheel. (4 marks)

ii. If the torque on the faster wheel is 75 N-m, find the axial thrust on each shaft, neglecting friction. (2 marks)

QUESTION FOUR (20 MARKS)

a) Depending upon the angles between the shafts and the pitch surfaces, State and briefly explain **FOUR** classifications of bevel gears. (2 marks)

b) Consider a pair of bevel gears in mesh, as shown in **Fig. Q4 (b).**

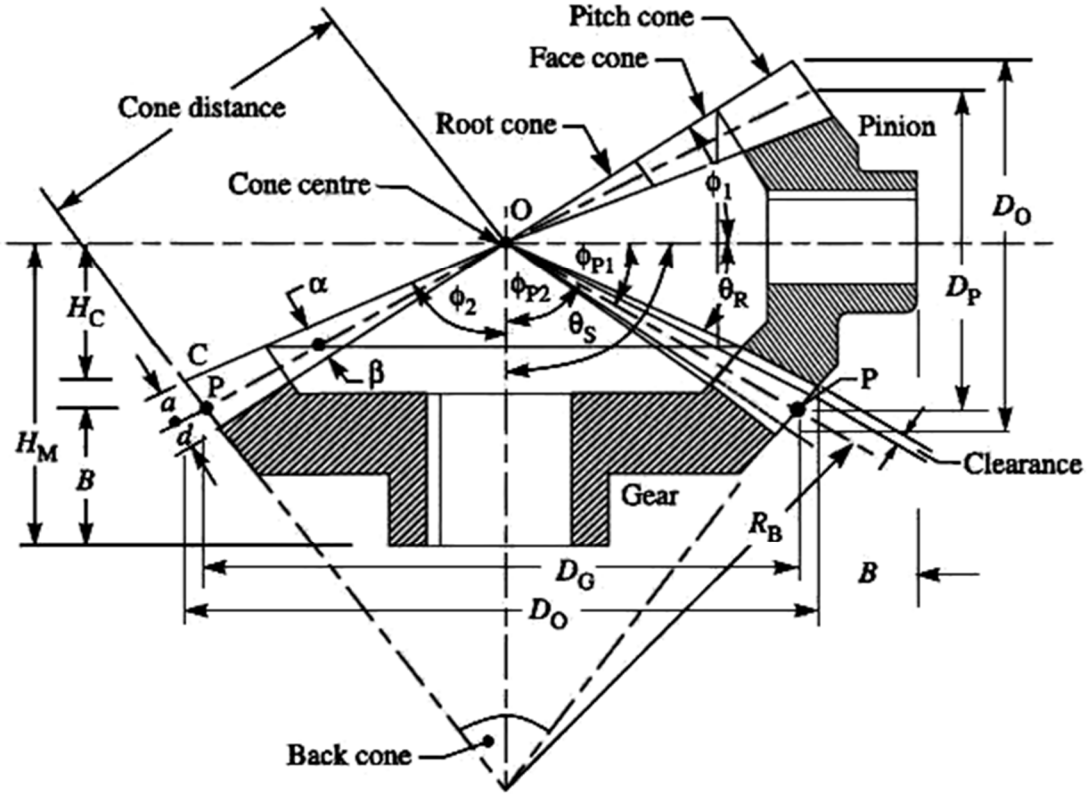

Fig. Q4 (b)

Letting θ_{P1} = Pitch angle for the pinion, θ_{P2} = Pitch angle for the gear, θ_S = Angle between the two shaft axes, D_P = Pitch diameter of the pinion, D_G = Pitch diameter of the gear, T_P = Number of teeth on the pinion, T_G = Number of teeth on the gear, N_P = Speed of rotation of the pinion, N_G = Speed of rotation of gear and *V.R.* = Velocity ratio = $(D_G/D_P) = (T_G/T_P) = (N_P/N_G)$.

i. Show that $\theta_{P1} = \tan^{-1}\left(\frac{\sin\theta_S}{V.R+\cos\theta_S}\right)$ and $\theta_{P2} = \tan^{-1}\left(\frac{\sin\theta_S}{\frac{1}{V.R}+\cos\theta_S}\right)$ (3 marks)

ii. When the angle between the shaft axes is 90º (θ_S = 90º), write other expressions for θ_{P1} and θ_{P2}. (1 mark)

c) A 35 kW motor running at 1200 r.p.m. drives a compressor at 780 r.p.m. through a 90° bevel gearing arrangement. The pinion has 30 teeth. Determine; Velocity ratio, Number of teeth on the gear, T_G, θ_{P1}, θ_{P2}, Formative number of teeth for pinion, Formative number of teeth for the gear, Tooth form factor for the pinion and tooth form factor on the gear. The form factor for teeth may be taken as $0.124 - \frac{0.686}{T_E}$ where T_E is the number of teeth equivalent of a spur gear. (4 marks)

d) With neat sketches and brief explanations, state **TWO** types of worm and **TWO** types of worm gears. (5 marks)

e) A triple threaded worm has teeth of 6 mm module and pitch circle diameter of 50 mm. If the worm gear has 30 teeth of 14½° and the coefficient of friction of the worm gearing is 0.05. Find:

i. The lead angle of the worm. (1 mark)

ii. Velocity ratio. (1 mark)

iii. Centre distance. (1 mark)

iv. Efficiency of the worm gearing. (2 marks)

QUESTION FIVE (20 MARKS)

a) An epicyclic train of gears is arranged as shown in **Fig. Q5 (a).** How many revolutions does the arm, to which the pinions B and C are attached, make:

i. When A makes one revolution clockwise and D makes half a revolution anticlockwise. (5 marks)

ii. When A makes one revolution clockwise and D is stationary. (5 marks)

The number of teeth on the gears A and D are 40 and 90 respectively.

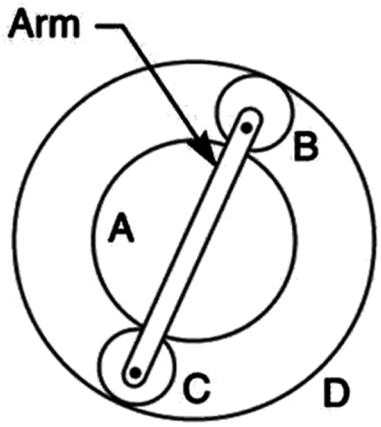

Fig. Q5 (a)

b) A pair of gears having 40 and 20 teeth respectively are rotating in mesh, the speed of the smaller being 2000 RPM. Determine the velocity of sliding between the gear teeth faces at the point of engagement, at the pitch point, and at the point of disengagement if the smaller gear is the driver. Assume that the gear teeth are 20^0 involute form, addendum length is 5 mm and the module is 5 mm. Also, find the angle through which the pinion turns while any pairs of teeth are in contact. (10 marks)