



MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

SECOND YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE IN TELECOMMUNICATION ENGINEERING

SPH 205: MATHEMATICAL PHYSICS

DATE: 11/12/2019

TIME: 8.30-10.30 AM

INSTRUCTIONS TO CANDIDATES

Answer question ONE (COMPULSORY) and any other TWO questions.

Question one carries 30 marks, all other questions carry 20 marks each

QUESTION ONE (30 MARKS)

- a) A force of $\vec{F} = (2.0\hat{i} + 3.0\hat{j})N$ is applied to an object that is pivoted about a fixed axis aligned along the z-coordinate axis. The force is applied at a point located at $\vec{r} = (4.0\hat{i} + 5.0\hat{j})m$. Find the torque $\vec{\tau}$ applied to the object. (3 marks)
- b) Find the area of a parallelogram with sides $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 4\hat{i} + 5\hat{j} + 6\hat{k}$. (2 marks)
- c) Find the volume V of the parallelepiped with sides $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 4\hat{i} + 5\hat{j} + 6\hat{k}$ and $\vec{c} = 7\hat{i} + 8\hat{j} + 10\hat{k}$. (2 marks)
- d) A particle moving in the xy plane undergoes a displacement given by $\Delta\vec{r} = (2.0\hat{i} + 3.0\hat{j})m$ as a constant force $\vec{F} = (5.0\hat{i} + 2.0\hat{j})N$ acts on the particle. Calculate the work done by $\vec{F}$ on the particle. (3 marks)
- e) i) Find the gradient of the scalar field; $f(x, y, z) = xy^2 - yz$ (3 marks)
- ii) Find the divergence of the vector field $\vec{F} = 3x^2\hat{i} - 6xy\hat{j}$. Explain the result (2 marks)

- iii) Find the curl of the vector field $F = y^3\hat{i} + xy\hat{j} - z\hat{k}$. (3 marks)
- f) The first three terms of an infinite geometric progress are; 16, 12 and 9.
- i. Write down the common ratio (1 mark)
- ii. Find the sum of the terms of the progression (2 marks)
- g) Evaluate the limits; $\lim_{x \rightarrow 1} (x^2 + 2x^3)$, $\lim_{x \rightarrow 0} (x \cos x)$, $\lim_{x \rightarrow \pi/2} \left(\frac{\sin x}{x} \right)$ (3 marks)
- h) Find the first and second partial derivatives of the function $f(x, y) = 2x^3y^2 + y^3$. (3 marks)
- i) Show that $I \equiv \int_{-1}^1 U_n(x)U_n(x)(1-x^2)^{1/2} dx = \frac{\pi}{2}$. (3 marks)

QUESTION TWO (20 MARKS)

- a) Show that if $\vec{a} = \vec{b} + \lambda\vec{c}$ for some scalar λ , then $\vec{a} \times \vec{c} = \vec{b} \times \vec{c}$. (3 marks)
- b) Find the direction of the line of intersection of the two planes;
 $x + 3y - z = 5$ and $2x - 2y + 4z = 3$. (3 marks)
- c) Find the vector area of the surface of the hemisphere $x^2 + y^2 + z^2 = a^2$, $z \geq 0$, by evaluating the line integral $S = \frac{1}{2} \oint_C \vec{r} \times d\vec{r}$ around its perimeter. (6 marks)
- d) i) Find the greatest number of terms required for the sum of; $4+9+14+19+\dots$ to exceed 2000. (3 marks)
- ii) State the mathematical form of a power series. Determine the range of values of x for which the following power series converges: $P(x) = 1 + 2x + 4x^2 + 8x^3 + \dots$ (5 marks)

QUESTION THREE (20 MARKS)

- a) A particle of mass m with position vector $\vec{r}$ relative to some origin O experiences a force $\vec{F}$, which produces a torque (moment) $\vec{\tau} = \vec{r} \times \vec{F}$ about O . The angular momentum of the particle about O is given by $\vec{L} = \vec{r} \times m\vec{V}$, where $\vec{V}$ is the particle's velocity. Show that the rate of change of angular momentum is equal to the applied torque. (3 marks)
- b) Show that $\nabla \cdot (\nabla \phi \times \nabla \phi) = 0$, where ϕ and ϕ are scalar fields. (4 marks)

- c) Find the vector area of the surface of the hemisphere $x^2 + y^2 + z^2 = a^2$ with $z \geq 0$.
(6 marks)
- d) Plane polar coordinates, (ρ, ϕ) and Cartesian coordinates, (x, y) are related by the expressions; $x = \rho \cos \phi$, $y = \rho \sin \phi$. An arbitrary function $f(x, y)$ can be re-expressed as a function $g(\rho, \phi)$. Transform the expression $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$ into one in (ρ, ϕ) .
(7 marks)

QUESTION FOUR (20 MARKS)

- a) Find the element of the area on the surface of a sphere of radius a , and hence calculate the total surface area of the sphere.
(7 marks)
- b) For a compressible fluid with time-varying position-dependent density $\rho(r, t)$ and velocity field $v(r, t)$, in which fluid is neither being created nor destroyed, show that; $\frac{\partial \rho}{\partial t} + \Delta(\rho \cdot v) = 0$.
(6 marks)
- c) Express the vector field $\vec{a} = yz\hat{i} - y\hat{j} + xz^2\hat{k}$ in cylindrical polar coordinates, and hence calculate its divergence. Show that the same result is obtained by evaluating the divergence in Cartesian coordinates.
(7 marks)

QUESTION FIVE (20 MARKS)

- a) Show that the area of a region R enclosed by a simple closed curve C is given by $A = \frac{1}{2} \oint_C (x dy - y dx) = \oint_C x dy = - \oint_C y dx$. Hence calculate the area of the ellipse $x = a \cos \phi$, $y = b \sin \phi$.
(5 marks)
- b) A ladder of length, L stands on level ground and can be leaned at any angle against a vertical wall. Find the equation of the curve bounding the vertical area below the ladder.
(5 marks)
- c) Find the angle between the vectors; $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$.
(3 marks)
- d) Given the vector field $\vec{a} = y\hat{i} - x\hat{j} + z\hat{k}$, verify Stoke's theorem for the hemispherical surface $x^2 + y^2 + z^2 = a^2$, $z \geq 0$.
(4 marks)
- e) The Cartesian coordinates of a point in the xy plane are; $(x, y) = (-3.50, -2.50)\text{m}$. Find the polar coordinates of this point.
(3 marks)