MACHAKOS UNIVERSITY

University Examination 2018/2019

SCHOOL OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONICS ENGINEERING)

EEE 415: ELECTRODYNAMICS AND INSULATING MATERIALS

DATE: 17/5/2019 **TIME: 8.30-10.30 AM**

INSTRUCTIONS

Answer Question ONE and any other TWO Questions.

Important Physical Constants

$\mu_0 = 4\pi x 10^{-7} H/m$

$\epsilon_0 = 8.854 x 10^{-12} F/m$

QUESTION ONE (30 MARKS) (COMPULSORY)

a) Show that the two "Divergence" Maxwell's Equations are implied by the two "Curl" Equations and the Equation of Continuity (4 marks)

b) A plane wave is propagating in a lossy media. Beginning from the Maxwell's Equations, show that the complex power (P_s)delivered by the source $\vec{J_s}$ consists of the averages of the complex pointing vector , the conduction losses, dielectric losses and the net reactive energy stored in the fields (6 marks)

c) An *Elliptically Polarized* wave propagating in the +z direction is represented by the x and y components of the electric field vector given by

$$E_x(z,t) = E_1 \sin(\omega t - \beta z)$$

$$E_y(z,t) = E_2 \sin(\omega t + \delta - \beta z)$$

i. Demonstrate the generation of *Circular* and *Linear* Polarization from the *Elliptical Polarization*. All the symbols carry their useful meaning (4 marks)

ii. Show that the *Poynting Vector* of the Elliptical Polarization is given by

$$\vec{S}_{ave} = \hat{z}\frac{\eta_0}{2}H^2 = \hat{z}\frac{1}{2\eta_0}E^2$$

All the symbols carry their useful meaning (4 marks)

iii. Given that

$E = 10\,\text{Cos}\left(\omega t + \frac{4\pi}{3}z\right)U_x V/m$ and that

$H = \frac{U_z x E}{\mu_0}\ A/m$

Determine the *average power per unit area* transported by the wave at a frequency of 500MHz (3 marks)

d) The current through a gap filled with a gaseous insulating medium can be expressed by

$$I = \frac{I_0 exp\ (\alpha_0 d)}{\{1 - \alpha_s(exp\ (\alpha_0 d) - 1)\}}$$

All the symbols carry their usual meaning

i. Derive the *Townsend Breakdown Criterion* hence explain the three cases for such a criterion (3 marks)

ii. Derive the *Panchen's Law* from Question 1 d i) above (2 marks)

iii. Given that $\alpha_0 d = K$ as the general form of Townsend Criterion, and that

1. $\frac{\alpha_0}{p} = A\exp\left(-\frac{Bp}{E}\right)$

Show that the point of minimum potential V_B can be given by

$$V_B = \left(\frac{E}{p}\right)pd + \sqrt{\frac{K}{C}}\sqrt{pd}$$

A,B,C and E are empirical constants while K defines the relationship between the Townsends and the Panchen's Law (5 marks)

QUESTION TWO (20 MARKS)

a) Beginning from the Maxwell's Equations, show that (3 marks)

Show that

$$\nabla^2 E - \gamma^2 E = 0$$

Where

$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\epsilon}\left[1 + \frac{\sigma}{j\omega\epsilon}\right]^{1/2}$$

All the symbols carry their usual meaning

b) Given that $\epsilon = \epsilon' - j\epsilon''$, show that (7 marks)

$$\alpha = \omega\sqrt{\frac{\mu\epsilon'}{2}}\left[\sqrt{1 + \left(\frac{\epsilon''}{\epsilon'}\right)^2} - 1\right]^{1/2}$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon'}{2}}\left[\sqrt{1 + \left(\frac{\epsilon''}{\epsilon'}\right)^2} + 1\right]^{1/2}$$

c) For a low loss dielectric, derive the propagation constant , phase and group velocity for a signal propagating through the medium (6 marks)

d) A narrow-band signal propagates in a lossy dielectric medium with a loss tangent of 0.2 and 550 kHz is the carrier frequency of the signal. If the dielectric constant of the medium is 2.5, determine the (4 marks)

i. Propagation Constant

ii. Group Velocity.

QUESTION THREE (20 MARKS)

a) Derive the Maxwell's Equation Originating from the following Laws

i. Gauss Law

ii. Ampere's Law (3 marks)

b) Using the Maxwel's Equations in Question 2a) above, derive the *Equation of Continuity* (3 marks)

c) Charges are introduced into the interior of a copper conductor during the time $t < 0$. Determine the time it takes these charges to move to the surface of the conductor so that both the interior charge density and the interior electric field are zero (6 marks)

d) The current density in a material is defined by the relation

$$J = \exp x^2\, U_x$$

Find the time rate of increase of the charge density at $x = 1$ (2 marks)

e) The current density in a certain region can be approximated in spherical coordinates as

$$J = J_0 \frac{\exp -t/\tau}{r} U_r$$

i. Determine the total current that leaves a spherical surface whose radius is a at the time $t = \tau$ (2 marks)

ii. Derive the expression of the charge density $\rho(r, t)$ (3 marks)

QUESTION FOUR (20 MARKS)

a) For a source –free polarized medium, the Electric Hertzian Potential $\vec{\Pi}_e$ can be defined such that the magnetic field intensity $\vec{H}$ is given by

$$\vec{H} = j\omega\epsilon_0 \vec{\nabla} x \vec{\Pi}_e$$

If $\vec{P}$ is the volume density of polarization in the medium, express the Electric Field Intensity $\vec{E}$ in terms of $\vec{\Pi}_e$ and $\vec{P}$ (4 marks)

b) Show that $\vec{\Pi}_e$ satisfies the Inhomogeneous Helmholtz Equation (7 marks)

$$\nabla^2 \vec{\Pi}_e + k^2 \vec{\Pi}_e = -\frac{\vec{P}}{\epsilon_0}$$

Where $k = \omega\sqrt{\mu_0 \epsilon_0}$ and all the other symbols carry their usual meaning.

c) The Electric Field intensity of a linearly polarized uniform wave propagating in the +z direction in sea water at the surface $z = 0$ is

$$\vec{E} = \hat{x} 100 \cos(\pi x 10^7) V/m$$

Sea water has $\epsilon_r = 72$, $\mu_r = 1$ and $\sigma = 4 S/m$.

Determine the expression for $\vec{E}(z, t)$ and $\vec{H}(z, t)$ at $z = 0.8m$ as a function of time (9 marks)

QUESTION FIVE (20 MARKS)

a) A parallel polarized wave at an oblique incidence on a dielectric interface is as shown in Figure Q 5a). Derive the expressions for the *Brewster and Transmission Angles*. All symbols carry their usual meaning (7 marks)

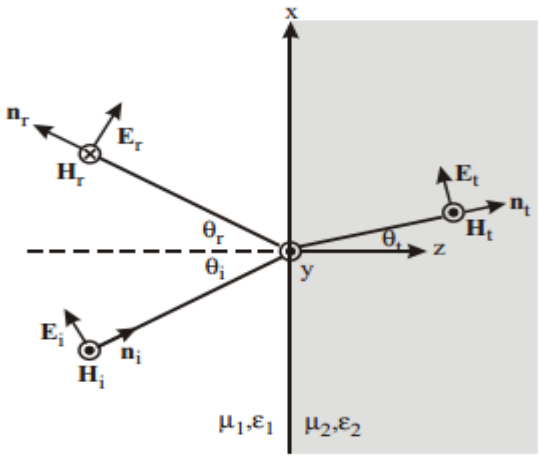

Figure Q 5a)

b) The same wave in Question 5a) is perpendicularly polarized. Derive the expressions for the Fresnel's Coefficients of Reflection and Transmission. All symbols carry their usual meaning (7 marks)

c) The dielectric constant of pure water is 80. Determine the following

i. Brewster Angle and Angle of Transmission for Question 5a) above (3 marks)

ii. Reflection and Transmission coefficients at $\theta_i = \theta_{Bp}$ (3 marks)