MACHAKOS UNIVERSITY

ISO 9001:2008 Certified

School of Education for the Bachelor's Degree

Second year examinations Second semester November 2017

SMA 203 LINEAR ALGEBRA II

Department of Mathematics and Statistics.

Time 2hours

Attempt question ***one (compulsory)*** and any other ***two questions***.

Question 1(30mks)

a.) Let V be the vector space of all functions from the real field R into R. Show that W is a subspace of V given that $W = \{ f : f(3) = 0 \}$ i.e. consists of those functions which map 3 onto 0. (3mks)

b.) Express $(2, -5, 3)$ as a linear combination of $(2, -3, 2)\,(2, -4, 1)$ and $(1, -5, 7)$ (3mks)

c.) Determine the value of h such that $(1, 3, 3)\,(-2, -6, h)\,(3, 9, -1)$ are $L..D$ (4mks)

d.) Let $V = P_3(x) = \{set\ of\ all\ polynomials\ of\ \deg ree \leq 3\}$ a basis for V is 1, x, x^2, x^3.

Define $D : V \to v$ where $D\,f(x) = \dfrac{d\,f(x)}{dx}$. D is the differentiation mapping. D is linear.

Determine the matrix of D. (5mks)

e.) Define $f : R \to R$ by $f(x) = \sin x$. prove that $f(x)$ is not linear. (5mks)

f.) Define $f : M_{(2,2)} \to M_{(2,2)}$ by $f(x) = Ax - xA$, where A is a fixed 2×2 matrix. Determine nullity and rank of $f(x)$. Given that $A = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$. (5mks)

g.) Diagonalize the matrix (5mks)

$$\begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

Question 2(20mks)

a.) Define $f : R^3 \to R^2$ by $f(x\ ,\ y\ ,\ z) = (\,x,\ x\ )$. Determine;

i) rank f (2mks)

ii) nullity f (3mks)

b.) Let $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix}$ be the matrix of $R^3 \to R^3$ relative to the standard basis of R^3. Calculate the matrix of f relative to the basis $(1,\ 1,\ 1),\ (1,\ 2,\ 1),\ (0,\ -1,\ 1)$. (5mks)

c.) Suppose that a linear Mapping f has matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 5 & 3 \\ 5 & 14 & 9 \\ 7 & 18 & 11 \end{bmatrix}$$

With respect to standard basis. Determine;

i) a basis for imf (5mks)

ii) a basis for $\ker f$ (5mks)

Question 3(20mks)

a.) Let $V = R^4$; $w = \{x, y, z, w \,/\, x + y - z = 0\}$. Show that w is a subspace of R^4 (5mks)

b.) Let v be a subspace of R^4 spanned by $(1\ ,\ 0,\ \ 2,\ 1)\,(1\ ,\ -1\ ,\ 1\ ,\ 2)\ \ (0,\ 0,\ 3,\ 1)$ and u be the subspace of R^4 spanned by $(2\ ,\ -1,\ 1,\ \ 2\ )\,(0\ ,\ 1\ ,\ 1\ ,\ 0\ )(1\ ,\ 2\ ,\ 1,\ 0\ )$. Determine a basis for;

i) $u + v$ (6mks

ii) $u \cap v$ (6mks)

c.) Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. Determine the characteristic polynomial of A. (3mks)

Question 4 (20mks)

a.) Consider the matrix $A = \begin{pmatrix} -1 & -5 \\ 10 & 14 \end{pmatrix}$. Work out the matrices P *and* P^{-1} such that $P\ A\ P^{-1} = D$. Where D is a diagonal matrix. (6mks)

b.) Let $A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -2 & 4 \end{bmatrix}$ Work out the minimal polynomial of A (7mks)

c.) Calculate the eigen values and the eigen vectors of the matrix

$$A = \begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix}$$ (7mks)

Question 5(20mks)

a.) Let C denote a set of complex numbers $C = (a + bi),\ a, b \in R,\ i = \sqrt{-1}$. show that C is a vector space over R (4mks)

b.) Let V be a vector space and let w be a fixed vector in V . Define
$W = \{set\ of\ all\, vectors\ in\ V\ which\ are \perp W\}$
$v_1 \perp v_2 \Leftrightarrow v_1\ is\ perpendicular\ to\ v_2$.
$w = \{v \in v\ /\ v \cdot w = 0\}$. Show that w is a subspace of v . (7mks)

c.) Define $f : R^3 \to R^3$ by $f(x, y, z) = (x + y, z, -x)$ let $s = \{(1, 1, 1), (1, 2, 1), (0, -1, 1)\}$ be an ordered basis for R^3. Determine the matrix of f relative to the basis s. (9mks)