# MACHAKOS UNIVERSITY

**University Examinations 2020/2021 Academic Year**

**SCHOOL OF PURE AND APPLIED SCIENCES**

**DEPARTMENT OF MATHEMATICS AND STATISTICS**

**FIRST YEAR FIRST SEMESTER EXAMINATION FOR**

**BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)**

**BACHELOR OF SCIENCE (MATHEMATICS)**

**BACHELOR OF EDUCATION**

**BACHELOR OF ARTS**

**SMA 102: BASIC MATHS**

**DATE: 24/2/2021** **TIME: 2.00-4.00 PM**

***INSTRUCTIONS:***
***Answer Question One and Any Other Two Questions***
***Programmable calculators are prohibited***

**QUESTION ONE (30 MARKS) (COMPULSORY)**

a) In a recent survey of 400 students in a school 100 were listed as smokers and 150 as chewers of gum; 75 were listed as both smokers and gum chewers. Find how many students were neither smokers nor chewers of gum. (4 marks)

b) Solve the inequality: $(x-6)(x+1) > 2x - 12$ . (5 marks)

c) When $x^3 - x^2 + ax + b$ is divided by $(x-1)$ the remainder is $-8$ and when the same expression is divided by $(x+1)$ the remainder zero. Calculate the values of $a$ and $b$, and factorize the expression completely. (5 marks)

d) Determine the product of the complex numbers $Z_1 = \frac{1}{2} + \frac{1}{2}i$ and $Z_2 = -\frac{1}{4} - \frac{3}{4}i$ , then show on an argand diagram the radius vector corresponding to this product. (5 marks)

e) A company wants to select a team of 8 workers for its repair operations. There are only 6 women and 15 men with this expertise. In how many ways can the selection be done if at least five men are to be in the team? (4 marks)

f) Determine whether the following is a tautology, a contradiction or neither by constructing a truth table. $(P \Longrightarrow Q) \wedge (\sim P \vee Q)$ (4 marks)

g) Use mathematical induction to prove that: $1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$ (3 marks)

**QUESTION TWO (20 MARKS)**

a) Describe each of the following terms as used in set theory giving an example in each case:

i. Symmetric difference sets $A$ and $B$. (3 marks)

ii. The power set of a set. (3 marks)

iii. Product of sets. (3 marks)

b) In a survey of the dinning preferences of 110 university resident students at the end of a semester, the following facts were discovered about *Adams Hotel* ($A$), *Brothers Pizza* ($B$), and *University Cafeteria* ($C$).

30 liked A but not B.

21 liked A only.

63 liked A

58 liked B

25 liked B and A but not C

18 liked B and C.

25 liked home cooking (i.e. disliked all three)

i. Draw a Venn diagram that represents this survey and label all the areas. (3 marks)

ii. How many liked A or C? (3 marks)

iii. How many liked all three? (3 marks)

iv. How many liked only C? (2 marks)

**QUESTION THREE (20 MARKS)**

a) Express with real denominator: $Z = \frac{1}{2+3i} + \frac{1}{2-3i}$ . (4 marks)

b) The roots of the equation $z^2 - z + 1 = 0$ are $z_1 = r_1(cos\theta_1 + isin\theta_1)$ and $z_2 = r_2(cos\theta_2 + isin\theta_2)$. Find $r_1,\ r_2,\ \theta_1,$ and $\theta_2$ (6 marks)

c) Find the square roots of $-5 + 12i$ by equating the real and imaginary parts of

$$(a + bi)^2 = -5 + 12i$$ (5 marks)

d) Determine the locus given by $Z\bar{Z} + 2(Z + \bar{Z}) = 0$, where $Z = x + iy$ and $\bar{Z}$ is its complex conjugate. (5 marks)

**QUESTION FOUR (20 MARKS)**

a) Find the number of distinguishable ways of arranging the letters in the word.

**MISCELLANEOUS** (4 marks)

b) Show that $(3x + 1)$ is a factor of the expression $3x^3 + 4x^2 + 7x + 2$. (4 marks)

c) Find the values of $a$ and $b$ if $(x - 2)$ and $(x + 1)$ are both factors of $ax^3 + 3x^2 - 9x + b$ and state the third factor of the expression. (6 marks)

d) Determine the first four terms of the expansion of $(1 + x)^{21}$ in ascending powers of $x$. Hence find the coefficient of $x^3$ in the expansion of $(2 - 3x)(1 + x)^{21}$. (6 marks)

**QUESTION FIVE (20 MARKS)**

a) Use Element argument method or Formal proof to show that $A \backslash B = A \cap B^c$. (4 marks)

b) Prove by mathematical induction that:

$(1 \times 2) + (2 \times 3) + (3 \times 4) + \cdots + [n(n + 1)] = \frac{1}{3}n(n + 1)(n + 2)$ (4 marks)

c) Simplify: $(-1 + i)(1 - i\sqrt{3})$, leaving your answer in the form $z = r(cos\theta + isin\theta)$, where $\theta$ is the principal value of the argument. Deduce the value of $\sqrt[3]{z^2}$. (6 marks)

d) Find the range of values of $x$ for which $3x + 4 < x^2 - 6 < 9 - 2x$. (6 marks)