MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (PROGRAMMING AND STATISTICS)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION

BACHELOR OF ARTS

SMA 203: LINEAR ALGEBRA II

DATE: 10/12/2021 **TIME: 8:30 – 10:30 AM**

INSTRUCTION:
Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) State the conditions for a linear transformation to have an inverse (2 marks)

b) Evaluate the following by inspection giving reasons for your answer:

i. $$\begin{vmatrix} 1 & 0 & 0 & 0 \\ -9 & -1 & 0 & 0 \\ 12 & 7 & 8 & 0 \\ 4 & 5 & 7 & 2 \end{vmatrix}$$

ii. $$\begin{vmatrix} 3 & -1 & 2 \\ 6 & -2 & 4 \\ 1 & 7 & 3 \end{vmatrix}$$ (4 marks)

c) Suppose that $C = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ -1 & -2 & 2 \end{bmatrix}$ is the matrix representation of a linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$. Find the rank and nullity of T. (5 marks)

d) Let $f: \mathbb{R}^3 \to \mathbb{R}^3$ be the linear mapping defined by

$$f(x, y, z) = (2x + y - z, -x - y - z, x - 2y + z)$$

and let $S = \{(1,1,1), (2,2,0), (3,0,0)\}$ be a basis for $\mathbb{R}^3$. Find the matrix A of f and a matrix B similar to A. (5 marks)

e) Define a linear mapping $f: \mathbb{R}^2 \to \mathbb{R}^2$ by $f(x, y) = (x + y, x + y)$. Find a basis and the dimension of

i. Its image U (3 marks)

ii. Its Kernel W (4 marks)

f) Find a basis or bases for the eigenspace(s) associated with the matrix $E = \begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix}$

(5 marks)

g) State the Cayley-Hamilton Theorem (2 marks)

QUESTION TWO (20 MARKS)

a) Evaluate $|D| = \begin{vmatrix} -1 & 0 & -2 & 3 \\ 1 & 5 & -5 & 0 \\ 2 & -3 & 4 & -4 \\ 1 & 4 & -2 & 0 \end{vmatrix}$ (6 marks)

b) Consider the basis $S = \{v_1, v_2, v_3\}$ for $\mathbb{R}^3$, where $v_1 = (1,1,1), v_2 = (1,1,0), and\ v_3 = (1,0,0)$, and let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be the linear operator for which $T(v_1) = (2, -1,4)$, $T(v_2) = (3,0,1), T(v_1) = (-1,5,1)$. Find a formula for (x_1, x_2, x_3), and use the formula to find $T(2,4, -1)$. (5 marks)

c) If $f: \mathbb{R}^3 \to \mathbb{R}^3$ is multiplication by the matrix by the matrix $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 2 & 2 \\ 0 & 1 & 0 \end{bmatrix}$,

i. Show that f^{-1} exists (2 marks)

ii. Find A^{-1} (5 marks)

iii. Find formula for $f^{-1}(x, y, z)$ (2 marks)

QUESTION THREE (20 MARKS)

a) Consider the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

i. Determine the eigenvalues and bases for the corresponding eigenspaces of A.

(8 marks)

ii. Find the invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$

(9 marks)

iii. Use the diagonal matrix D in b) above to compute A^7 (3 marks)

QUESTION FOUR (20 MARKS)

a) Use row reduction to show that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (b-a)(c-a)(c-b)$$ (6 marks)

b) Use Cayley Hamilton theorem to find the inverse of $C = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$ (8 marks)

c) Find the minimal polynomial of $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (6 marks)

QUESTION FIVE (20 MARKS)

a) Let $f: U \longrightarrow V$ be a linear transformation from vector space U to a vector space V. Prove that f is one-to-one if and only if $N(f) = 0$. (4 marks)

b) Show that the linear mapping $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^4$ is not invertible. (2 marks)

c) Explain what it means by a diagonal matrix and give an example. (2 marks)

d) Find the minimal polynomial of $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (4 marks)

e) List down any four properties of determinants. (4 marks)

f) Given matrices $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$, determine if A and B are similar. (4 marks)