MACHAKOS UNIVERSITY COLLEGE

(A Constituent College of Kenyatta University)
University Examinations for 2015/2016 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST SEMESTER EXAMINATION FOR DEGREE IN BACHELOR OF SCIENCE IN MATHEMATICS

SMA360: MULTIVARIATE STATISTICAL METHODS I

Date: 5/08/2016 **Time: 8:30-10:30 AM**

INSTRUCTION:

Answer Question One and any other Two Questions

QUESTION ONE (30 MARKS)

(a) Differentiate between a random vector and a random matrix (2 marks)

(b) Consider the following data with three variables X, Y and Z

X	Y	Z
3	9	12
4	2	8
0	6	6
2	5	4
1	8	10

Calculate

(i) Mean vector (2 marks)

(ii) Covariance matrix (5 marks)

(iii)Correlation matrix (3 marks)

(c) Define the characteristic function of a random variable. Let $\boldsymbol{X} \sim N_p(\boldsymbol{\mu}, \Sigma)$ and define $\boldsymbol{Y} = \boldsymbol{AX} + \boldsymbol{b}$ where **A** is a $q \times p$ matrix of constants and **b** is a $q \times 1$vector of constants. Use characteristic technique to show that $\boldsymbol{Y} \sim N_q(\boldsymbol{A\mu} + \boldsymbol{b},\ \boldsymbol{A}\Sigma\boldsymbol{A}')$ (6 marks)

(d) Let $\boldsymbol{X} = (X_1, X_2, X_3)'$ be a random vector with mean vector $\boldsymbol{\mu} = (1, -1, 3)$ and covariance matrix $\Sigma = \begin{pmatrix} 4 & 1 & 2 \\ 1 & 1 & 0 \\ 2 & 0 & 2 \end{pmatrix}$. Find the mean vector and covariance matrix of a random vector $\boldsymbol{Y} = (Y_1, Y_2)'$ where $Y_1 = X_1 + X_2$ and $Y_2 = X_1 + X_2 - X_3$ (5 marks)

(e) Suppose that $\boldsymbol{X}' = (X_1, X_2, X_3)$ is a continuous random vector whose probability distribution is given by

$$f(x_1, x_2, x_3) = \begin{cases} 4x_1x_2x_3, & 0 < x_1 < x_2 < 1 \\ & 0 < x_3 < 2 \\ 0, & Otherwise \end{cases}$$

Obtain the

(i) Marginal joint distribution of X_2 and X_3 (3 marks)

(ii) Conditional distribution of $X_1 = x_1$ given that $X_2 = x_2, X_3 = x_3$ (2 marks)

(f) Write down the probability distribution of a multinomial random variable X (2 marks)

QUESTION TWO

(a) Write down a non-singular multivariate normal distribution. Hence prove that it is a probability distribution (10 marks)

(b) Suppose that $\boldsymbol{Z} = (X, \ Y)'$ is a bivariate random variable with probability distribution given by

$$f(x, y) = \begin{cases} \frac{6}{5}(x + y), & 0 \leq x \leq y \leq 1 \\ 0, & Otherwisw \end{cases}$$

Obtain

(i) Mean vector of **Z** (4 marks)

(ii) Covariance matrix of **Z** (6 marks)

QUESTION THREE

a) Let $\boldsymbol{X} = (X_1, X_2, \ldots, X_p)'$ be a p dimensional random vector with mean vector $\boldsymbol{\mu}$ and covariance matrix Σ. Let $\boldsymbol{Y}$ be another random variable defined on $\boldsymbol{X}$ as $\boldsymbol{Y} = \boldsymbol{AX}$ where **A** is a $q \times p$ matrix of constants. Obtain the mean vector and covariance matrix of $\boldsymbol{Y}$

(6 marks)

b) Let $\boldsymbol{X}' = (X_1, X_2, X_3)$ be a 3×1 random vector with mean vector $\boldsymbol{\mu}' = (2, 3, 1)$ and covariance matrix $\Sigma = \begin{pmatrix} 4 & 3 & -1 \\ 3 & 16 & 5 \\ -1 & 5 & 9 \end{pmatrix}$. Obtain mean vector and covariance matrix of

(i) Y=$X_1 + X_2 - X_3$ (3 marks)

(ii) $\boldsymbol{Y}' = (Y_1, Y_2, Y_3)$ where $Y_1 = X_1 + X_2 - X_3$, $Y_2 = X_1 + 2X_2 + X_3$ and $Y_3 = 2X_1 - X_2 + 2X_3$ (5 marks)

c) State and prove the weak law of large numbers (6 marks)

QUESTION FOUR

a) Suppose that a p-dimensional random vector X is normally distributed with mean vector $\boldsymbol{\mu}$ and covariance matrix Σ. Obtain the moment generating function of **X.** (12 marks)

b) Suppose that Y is a multinomial random variable which takes values y_1, y_2, y_3, y_4 *and* y_5 with probabilities $P_1 = 0.18, P_2 = 0.23, P_3 = 0.16, P_4 = 0.27$ and $P_5 = 0.16$. Find the probability that $y_1 = 1, y_2 = 2, y_3 = 0, y_4 = 2$ *and* $y_5 = 0$ (3 marks)

c) State and prove Chebychev's inequality (5 marks)

QUESTION FIVE

a) Let $\boldsymbol{X}' = (X_1, X_2, X_3, X_4)'$ be normally distributed with mean vector $\boldsymbol{\mu} = (1,2,3,-2)'$ and covariance matrix $\Sigma = \begin{pmatrix} 4 & 2 & -1 & 2 \\ 2 & 6 & 3 & -2 \\ -1 & 6 & 5 & -4 \\ 2 & -2 & -4 & 4 \end{pmatrix}$. Partition X into sub-vectors $\boldsymbol{X}^1 = (X_1, X_2)$ and $\boldsymbol{X}^2 = (X_3, X_4)$. Obtain

(i) Conditional distribution of $\boldsymbol{X}^1$ given $\boldsymbol{X}^2$ (6 marks)

(ii) Partial regression coefficient of $\boldsymbol{X}^1$ given $\boldsymbol{X}^2$ (4 marks)

(iii) Partial correlation coefficient of $\boldsymbol{X}^1$ given $\boldsymbol{X}^2$ (4 marks)

b) Define a moment generating function of a random vector. Let $\boldsymbol{X} \sim N_p(\boldsymbol{\mu}, \Sigma)$ and define $\boldsymbol{Y} = \boldsymbol{AX} - \boldsymbol{b}$ where **A** is a $q \times p$ matrix of constants and **b** is a $q \times 1$vector of constants. Use moment generating function technique to show that $\boldsymbol{Y} \sim N_q(\boldsymbol{A\mu} - \boldsymbol{b},\ \boldsymbol{A}\Sigma\boldsymbol{A}')$

(6 marks)