



MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

.....YEAR SEMESTER SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN MATHEMATICS

BACHELOR OF ARTS

SMA 436: METHODS OF APPLIED MATHEMATICS II

DATE:

TIME:

INSTRUCTIONS: Answer QUESTION **ONE** and any other **TWO** QUESTIONS

QUESTION ONE (30 MARKS) (COMPULSORY)

- a) Given that the $\delta_j^i = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$ Expand $\delta_j^i x_i x_j$. (3 marks)
- b) If a_{ij} are constants and $\delta_{pq} = \frac{\partial x_p}{\partial x_q}$, calculate the partial derivative $\frac{\partial}{\partial x_k} (a_{ij} x_i x_j)$. (3 marks)
- c) Define the following terms
- Umbral index (1 mark)
 - Covariant tensor of rank two (2 marks)
- d) Find the resolvent kernel associated with $k(x, t) = e^{x-t}$. (5 marks)
- e) Prove that a symmetric tensor of rank two has only $\frac{1}{2}n(n+1)$ different components in n-dimensional space. (4 marks)
- f) Define the Christoffel symbols of first and second kind. (2 marks)
- g) i. Define Fredholm integral equation of the second kind and give its general form (2 marks)
- ii. Solve the Fredholm equation $u(x) = 1 + \int_0^1 xu(t)dt$ using successive approximation method with initial condition $u_0(x) = 1$ (5 marks)

- h) Define a weakly singular integral equation. (3 marks)

QUESTION TWO (20 MARKS)

- a) State the Quotient law. (2 marks)
- b) Show that the expression $A(i, j, k)$ is a covariant tensor of rank three if $A(i, j, k)B^k$ is a covariant tensor rank two and B^k is a contravariant tensor (6 marks)
- c) Write down the laws of transformation of the following tensors
- i. A_{ij} (2 marks)
- ii. C_{lm}^{ijk} (2 marks)
- d) Show that $g_{ij}dx^i dx^j$ is an invariant (6 marks)

QUESTION THREE (20 MARKS)

- a) Prove that the metric tensor g_{ij} is a covariant symmetry tensor of rank two (12 marks)
- b) If $ds^2 = (dr)^2 + r^2(d\theta)^2 + r^2 \sin^2 \theta (d\phi)^2$ find the value of:
- i. $[22,1]$ and $[13,3]$ (4 marks)
- ii. $\begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$ and $\begin{Bmatrix} 3 \\ 1 \end{Bmatrix}$ (4 marks)

QUESTION FOUR (20 MARKS)

- a) Given $x^i = (x^1, x^2, x^3)$ and $\bar{x}^i = (\bar{x}^1, \bar{x}^2, \bar{x}^3)$ are coordinates of a point in Cartesian and cylindrical coordinate systems respectively, find the metric, component of the first and second fundamental tensor in cylindrical Coordinates. (15 marks)
- b) Use the method of successive approximation to solve the Fredholm equation
 $u(x) = \sin x + \int_0^{2\pi} \sin x \cos u(t) dt$ with initial condition $u_0(x) = 1$ (5 marks)

QUESTION FIVE (20 MARKS)

- a) Solve the Volterra integral equation $\phi(x) = x - \int_0^x (x-t)\phi(t) dt$ with the initial condition $\phi_0(x) = 0$ (10 marks)
- b) Solve the boundary value problem $y'' + y = F(x), y(0) + y'(0) = 0;$
 $y(\pi) + y'(\pi) = 0$ and find the Green's function for this problem. (10 marks)