MACHAKOS UNIVERSITY

University Examinations for 2020/2021

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF ARTS

SMA 431: DIFFERENTIAL GEOMETRY

DATE: 9/8/2021 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS: Answer Question One and Any Other Two Questions

QUESTION ONE(COMPUSARY) (30 MARKS)

a) A straight line passes through the points $A(2,3,4)$ and $B(5,1,6)$

i. Write down the direction vector of $\overrightarrow{AB}$ (2 marks)

ii. State the vector equation of the line AB (2 marks)

iii. State the Cartesian equation of the line AB (2 marks)

iv. State the parametric equation of the line AB (2 marks)

b) Determine the values of a so that $\vec{A} = ai - 2j + k$ and $\vec{B} = 2ai + aj - 4k$ are orthogonal. (3 marks)

c) Find the equation of a plane passing through the point $(2,1,-3)$ given that it is perpendicular to the vector $5i + 6j + 7k$ (3 marks)

d) Find $\nabla \circ (\nabla \phi)$ where $\phi = \phi(x, y, z)$ is a differentiable scalar function (5 marks)

e) If $\vec{A} = 3i - 2j + 4k$ and $\vec{B} = 2i - 3j - 2k$ determine the angle θ between the and the area of the triangle formed by vectors $\vec{A}$ and $\vec{B}$. (5 marks)

f) Given the space curve $x = 2t,\ y = t^2$ and $z = \frac{2}{3}t^3$

Find; Curvature k and Torsion τ (6 marks)

QUESTION TWO (20 MARKS)

a) The vectors $\vec{a} = 2i - j + k$, $\vec{b} = i + 2j - 3k$ and $\vec{c} = 3i - pj + 5k$ are coplanar. Find constant p

(3 marks)

b) For a space curve

$\vec{r} = l\cos ti + l\sin tj + mtk$ where l and m are constants.

Determine the following

i. The unit tangent $\vec{T}$ (3 marks)

ii. The curvature k (3 marks)

iii. The unit normal $\vec{n}$ (3 marks)

iv. The torsion τ and the radius of torsion σ (4 marks)

c) Prove the following Frenet-Serret formulae

$$\frac{d\vec{T}}{ds} = k\vec{n}$$

(4 marks)

QUESTION THREE (20 MARKS)

a) Find the equation of the plane determined by the points $A(2,-1,1)$, $B(3,2,-1)$ and $C(-1,3,2)$

(3 marks)

b) Find

i. A unit normal to the surface

$\phi(x, y, z) = xy - z^2 = 0$ at the point $(1,4,-2)$ (3 marks)

ii. The equation to the normal line (2 marks)

iii. The equation to the tangent plane at $(1,4,-2)$ (5 marks)

c) Find the second fundamental form on the surface $\vec{r} = ui + vj + (u^2 - y^2)k$ (7 marks)

QUESTION FOUR (20 MARKS)

a) Find the unit vector paralleled to the resultant of vectors

$\vec{r_1} = 2i + 4j - 5k$, and $\vec{r_2} = i + 2j + 3k$ (3 marks)

b) If $\vec{A} = (y^2 - x^2z^2)i + (x^2 + y^2)j - x^2yzk$, determine $curl\vec{A}$ at the point $(1,3,-2)$ (3 marks)

c) Find the length of the arc of the curve $\vec{r} = \cos ti + e^t \sin tj + e^t k$ from $0 \le t \le \pi$ (5 marks)

d) Determine the volume of the parallelepiped whose edges are given by $\vec{a} = 2i - 3j + 4k$, $\vec{b} = i + 2j - k$ and $\vec{c} = 3i - j + 2k$ (4 marks)

e) Prove the vector triple product $\vec{A}(\vec{B}x\vec{C}) = \vec{B}(\vec{A} \circ \vec{C}) - \vec{C}(\vec{A} \circ \vec{B})$ (5 marks)

QUESTION FIVE (20 MARKS)

Consider the surface $\vec{r} = ui + vj + \left(u^2 + v^2\right)k$

i. Find the vector in the direction of the tangent to the curve $u = u_o$ on the surface (2 marks)

ii. Determine the vector normal to the surface (4 marks)

iii. What is the unit normal vector? (2 marks)

Find the equation to the tangent plane at the point P on the surface where $u = -1$ and $v = 1$ (5 marks)

iv. Equation to the normal line at P (7 marks)