MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE),

BACHELOR OF SCIENCE (MATHEMATICS),

BACHELOR OF EDUCATION

SMA 332: METHODS OF APPLIED MATHEMATICS I

DATE: 23/3/2021 **TIME: 2.00-4.00 PM**

INSTRUCTIONS TO CANDIDATES
Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a) State the general second order linear partial differential equation (P.D.E.) (2 marks)

b) Hence state the three classes of that P.D.E. while giving an example equation of the canonical form in each class. (5 marks)

c) Write the Fourier Sine series expansion of a function $f(x)$ over $[0,\pi]$, stating the expression of the Fourier coefficient. (3 marks)

d) Transform the wave equation into a set of ordinary differential equations. (5 marks)

e) Determine the Laplace transform of $f(x) = x^2$. (2 marks)

f) Use the Laplace transforms to solve the equation $y' + 2y = 3x^2$, $y(0) = 1$. (5 marks)

g) Given that $u = u(x, y)$, apply the change of variables $\theta = \theta(x, y)$, $\eta = \eta(x, y)$ to express the following in terms of the variables θ, η :

i) The second order partial derivative u_{xx} (4 marks)

ii) The second order partial derivative u_{xy} (4 marks)

QUESTION TWO (20 MARKS)

a) The Laplace transform of a function $u = u(x,t)$ is $L\{u(x,t)\} = \int_0^\infty e^{-st} u(x,t)dt$. Determine the Laplace transform of $\frac{\partial^2 u}{\partial t^2}$. (7 marks)

b) Given the partial differential equation $u_{xx} + yu_{yy} + \frac{u_y}{2} = 0$, in the xy-plane:

i) Determine the region in the xy-plane where it is hyperbolic (2 marks)

ii) Obtain the canonical form associated with the hyperbolic case. (11 marks)

QUESTION THREE (20 MARKS)

a) Sketch the function $f(x) = x, -2 < x < 2$ whereby $f(x+4) = f(x)$ and hence determine the appropriate Fourier series expansion of the function. (8 marks)

b) Determine the inverse Laplace transform of $\frac{6}{(s^2+1)(s^2+9)}$ using the convolution theorem

(6 marks)

c) Solve by Laplace's method the initial value problem

$y'' + 2y' + 2y = 0,\ y(0) = 1,\ y'(0) = -1$ (6 marks)

QUESTION FOUR (20 MARKS)

a) Solve $u_t = \alpha u_{xx}$, $0 < x < \pi$ subject to: $\begin{matrix} u(0,t) = u(\pi,t) = 0,\ t > 0 \\ u(x,0) = f(x),\ 0 < x < \pi \end{matrix}$ by separation of variables method to obtain the particular solution when $f(x) = x$. (14 marks)

b) Show that the set of functions $\{\cos n\pi, \sin m\pi\}$ is orthogonal over the interval $[-\pi, \pi]$

(6 marks)

QUESTION FIVE (20 MARKS)

a) The displacement of a semi-infinite elastic beam is described

$$u_{tt} = a^2 u_{xx},\ x > 0, t > 0$$

by $u(0,t) = f(t),\ \lim_{x\to\infty} \frac{\partial u}{\partial x} = 0,\ t > 0$. Solve for $u(x,t)$ by the Laplace transform method.

$$u(x,0) = 0,\ \left.\frac{\partial u}{\partial t}\right|_{t=0} = 0,\ t > 0$$

(10 marks)

b) Find the characteristics of the equation $4u_{xx} + 12u_{xy} + 9u_{yy} - 9u = 9$ and reduce it to the appropriate canonical form. (10 marks)