



# MACHAKOS UNIVERSITY

University Examinations for 2021/2022

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (TELECOMMUNICATION AND INFORMATION TECHNOLOGY)

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (APPLIED PHYSICS AND TECHNOLOGY)

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

BACHELOR OF EDUCATION (SPECIAL NEEDS)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

BACHELOR OF ARTS

SMA 103: ANALYTICAL GEOMETRY

DATE: 26/1/2022

TIME: 2:00 – 4:00 PM

## INSTRUCTIONS:

Attempt question *one (compulsory)* and any other *two questions*.

## QUESTION ONE (COMPULSORY) (30 MARKS)

- a) A point  $R(x, y)$  divides the line through the points  $A(4, -9)$  and  $B(-5, 25)$  externally in the ratio  $3 : 2$ . Calculate the coordinates of  $R(x, y)$  (3 marks)
- b) Consider two lines  $2x + y = 2$  and  $x - 2y = 6$ . Calculate the equation of a line joining the point of their intersection and the point  $(4, 3)$  (5 marks)
- c) Prove that  $\sin(A + 60^\circ) \sin(A + 120^\circ) = \frac{\sin 3A}{4 \sin A}$  (4 marks)

- d) Determine the length of the tangent line from the point  $(5,7)$  to the circle  $x^2 + y^2 - 4x - 6y + 9 = 0$  (5 marks)
- e) A parabola  $y = ax^2 + bx + c$  passes through the points  $(1,0)$ ,  $(2,2)$ , and  $(3,10)$ . Determine its equation. (5 marks)
- f) Work out the Cartesian equation for the curve  $r \cos\left(\theta - \frac{\pi}{3}\right) = 3$  (4 marks)
- g) Determine the polar equation of the ellipse  $x^2 + 9y^2 = 9$  (4 marks)

### QUESTION TWO (20 MARKS)

- a) Determine the equation of a circle through the points  $A(6,-1)$ ,  $B(-2,5)$ ,  $C(-6,-7)$ , sketch the circle (6 marks)
- b) Calculate the equation of the circle which is tangent to the x-axis and concentric with the circle  $2x^2 + 2y^2 - 11x + 6y - 8 = 0$  (6 marks)
- c) Determine the polar equation of the circle whose center has the Cartesian coordinates  $C(5,8)$  and radius  $r = 10\text{cm}$  (5 marks)
- d) Calculate the distance from the point  $(5,8)$  to the circle  $x^2 + y^2 + 2x + 6y + 3 = 0$  (3 marks)

### QUESTION THREE (20 MARKS)

- a) Solve for  $x$  in  $\sin 3x + \sin x = 0$  (4 marks)
- b) Determine the value of  $x$  for which  $2\cos x - 2\sqrt{3}\sin x = 3$  (5 marks)
- c) Show that
- i)  $\frac{\sin 4\theta + \sin 2\theta}{\cos 4\theta + \cos 2\theta} = \tan 3\theta$  (4 marks)
- ii)  $\sin 4\theta = 4\cos^3 \theta \sin \theta - 4\sin^3 \theta \cos \theta$  (3 marks)
- d) Given that  $\cos 2x = \frac{1}{8}$ , calculate the value of  $\cos x$  (4 marks)

### QUESTION FOUR (20 MARKS)

- a) Sketch the curve and label axes, foci and vertices described by the equation  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  (6 marks)
- b) Calculate the point on the parabola  $y^2 - 12y - 14x + 162 = 0$  through which the tangents which meet at  $Q(2,4)$  pass (6 marks)
- c) Determine the equation of the tangent to the parabola  $x^2 - 22x + 24y + 169 = 0$  at the point whose focal distance is 11 units (8 marks)

**QUESTION FIVE (20 MARKS)**

- a) Sketch the curve and label axes, foci and vertices described by the equation  
$$\frac{x^2}{25} - \frac{y^2}{16} = -1$$
 (12 marks)
- b) Calculate the angle between the lines  
 $y = 2x + 5$  and  $y = 12x - 1$  (4 marks)
- c) Calculate the normal equation of the line with  $\rho = 10 \text{ units}$  ,  $\omega = 60^\circ$  (4 marks)