MACHAKOS UNIVERSITY

University Examinations for 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

FIRST YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (TELECOMMUNICATION AND INFORMATION TECHNOLOGY)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF SCIENCE (APPLIED PHYSICS AND TECHNOLOGY)

SPT101/ SPH 103: MATHEMATICS FOR PHYSICS

MATHEMATICS FOR PHYSICS I

DATE: 26/8/2022 **TIME: 8.30-10.30 AM**

INSTRUCTIONS:

Answer QUESTION ONE, which is COMPULSORY, and **ANY OTHER TWO** questions.

Question 1 carries **30** marks and the others carry **20**marks each.

You may find this useful		
$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$	$\sin^2 x + \cos^2 x = 1$
$\int e^x\,dx = e^x + C$	$\int a^x\,dx = \frac{a^x}{\ln a} + C$	
$\int \sin x\,dx = -\cos x + C$	$\int \cos x\,dx = \sin x + C$	
$\int \sec^2 x\,dx = \tan x + C$	$\int \csc^2 x\,dx = -\cot x + C$	
$\int \sec x \tan x\,dx = \sec x + C$	$\int \csc x \cot x\,dx = -\csc x + C$	
$\int \sinh x\,dx = \cosh x + C$	$\int \cosh x\,dx = \sinh x + C$	
$\int \tan x\,dx = \ln\lvert \sec x\rvert + C$	$\int \cot x\,dx = \ln\lvert \sin x\rvert + C$	
$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$	$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C$	

QUESTION ONE (30 MARKS)

a) Find the range and the domain of the function $y=\sqrt{4-x^2}$ (**sketch a graph representing the domain and range**) (3 marks)

b) Find the roots for a quadratic equation $x^2+x+2=0$ in complex form (3 marks)

c) Solve for x in the equation $\log_2(x^3+5)=e^{\ln 5}$ (3 marks)

d) Find the first derivative $\left(\frac{dy}{dx}\right)$ of the function $y=\sin 3x^4$ (3 marks)

e) Using integration by parts, evaluate $\int x\sin x dx$ (4 marks)

f) Use partial fractions to decompose and evaluate $\int \frac{-x+4}{(x^2+1)(x-1)}dx$ (5 marks)

g) What is the probability of throwing a total of 6 points or less using three dices (3 marks)

h) Find the partial differentials $\frac{\partial R}{\partial x}$ for the function $R_{(x,y)}=\frac{x^2}{y^2+1}-\frac{y^2}{x^2+y}$ (3 marks)

i) Find the **angle** between vectors $\vec{A}=2\underset{\sim}{i}+2\underset{\sim}{j}-\underset{\sim}{k}$ and $\vec{B}=7\underset{\sim}{i}+24\underset{\sim}{k}$ (3 marks)

QUESTION TWO (20 MARKS)

a) Evaluate the integral $\int e^x \cos x dx$ (7 marks)

b) Evaluate $\int_0^2 \frac{9a^2da}{\sqrt[3]{a^3+8}}$ by substituting $u=a^3+8$. (**Write your answer to 3 significant figures**)

(7 marks)

c) Evaluate the trigonometric integral $\int \sin^4 x\cos^7 xdx$ (6 marks)

QUESTION THREE (20 MARKS)

a) A function F is defined for all real numbers t by the formula $F(t)=2(t-1)+4$. Evaluate F at the input value of 0, 2, $x+2$ and 3 (4 marks)

b) Use chain rule to evaluate $\frac{d}{dx}\left[\left(x^2+5\right)^3+3\right]^4$ (8 marks)

c) Obtain the second order derivative y'' of the function $y=xe^{2x^2}dx$ (8 marks)

QUESTION FOUR (TWENTY MARKS)

a) Graph the function $y = x^4 - 4x^3 + 10$ (8 marks)

b) Express $\frac{2x^3 - 4x^2 - x - 3}{x^2 - 2x - 3}$ as a sum of partial fractions and integrate (8 marks)

c) If 5 of 20 fuses in a box are, defective and 5 of them are randomly chosen for inspection, what is the probability that two of the chosen fuses are defective (4 marks)

QUESTION FIVE (TWENTY MARKS)

a) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for the following function $x^3 \sin\left(y^3\right) + xe^{3z} - \cos\left(z^2\right) = 3y - 6z + 2k$

(8 marks)

b) Express the product and quotient $\frac{z_1 z_2}{z_1}$ of the following complex numbers $z_1 = -2 + 3i$ and

$z_1 = 5 - i$ (6 marks)

c) Find $(\vec{A} \times \vec{B}) \times \vec{C}$, given that $\vec{A} = -\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$, $\vec{B} = \underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ and $\vec{C} = \underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}$ (6 marks)