



MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN COMPUTER SCIENCE

SCO 109: LINEAR ALGEBRA FOR COMPUTER SCIENCE

DATE: 22/3/2021

TIME: 2.00-4.00 PM

INSTRUCTIONS

Answer ALL the questions in Section A and ANY TWO Questions in Section B

SECTION A

QUESTION ONE (30 MARKS) (COMPULSORY)

- a) Find the dot product of the following vectors $(3, 4, -1)$ and $(4, 6, 1)$ (3 marks)
- b) Find the angle between the following vectors $3i + 4j + 6k$ and $i + 2j + k$ (4 marks)
- c) Solve the homogenous system by Gauss elimination method

$$x + 2y + z = 4$$

$$3x + 8y + 7z = 20$$

$$2x + 7y + 9z = 23 \quad (5 \text{ marks})$$

- d) If $\begin{vmatrix} 2y & 10 \\ 2y & 4y + 2 \end{vmatrix} = 4$ find y (3 marks)

Calculate the cross product of the vectors $\vec{u} = (1, 2, 3)$ and $\vec{v} = (-1, 1, 2)$. (3 marks)

- e) Find the inverse of the following matrix

$$\begin{bmatrix} 0 & 2 & 4 \\ 2 & 4 & 2 \\ 3 & 3 & 1 \end{bmatrix} \quad (4 \text{ marks})$$

- f) Show that the vector $(11, 3, -8)$ is a linear combination of the $(1, 1, 0)$ and $(2, 1, -1)$

- g) Calculate the x and y values for the vector $(x, y, 1)$ that is orthogonal to the vectors $(3, 2, 0)$ and $(2, 1, -1)$. (4 marks)
- h) Determine if $(1,2,3,1)$, $(2,2,1,)$ and $(-1,2,7,3)$ is linearly dependent (4 marks)

SECTION B

QUESTION TWO (20 MARKS)

- a) Find the equation of the plane passing through the point $(3, -1, 7)$ and perpendicular to the vector $\vec{n} = (4, 2, -5)$ (5 marks)
- b) Find the distance from the origin to the plane $2x + 3y - z = 2$ (5 marks)
- c) Find the parametric equations for the line of intersection of the plane $3x + 2y - 4z - 6 = 0$ and $x - 3y - 2z - 4 = 0$ (5 marks)
- d) Find the distance D between the point $(1, -4, -3)$ and the plane $2x - 3y + 6z = -1$ (5 marks)

QUESTION THREE (20 MARKS)

- a) Show that $w = \{ (x, y, z) | x + y + z = 0 \}$ is a subspace of $\mathbb{R}^3$ (5 marks)
- b) Calculate the value of k for the vectors $\vec{u} = (1, k)$ and $\vec{v} = (-4, k)$ knowing that they are orthogonal. (5 marks)
- c) Show that the vectors $u=(1,-1,0)$ $v=(1,3,-1)$ and $w=(5,3,-2)$ are linearly dependent. (5 marks)
- d) Find the rank of the following matrix

$$\begin{bmatrix} 2 & 4 & 3 \\ 3 & -3 & 2 \\ 2 & 0 & 3 \end{bmatrix} \quad (5 \text{ marks})$$

QUESTION FOUR (20 MARKS)

- a) Define a vector space (6 marks)
- b) Show that $W = \{ (x, y) / x = 2y \}$ is a subspace for $\mathbb{R}^2$. (4 marks)
- c) Prove that the diagonals of a rhombus are perpendicular. (5 marks)
- d) Find the parametric and the symmetric equations of the line passing through the point $(2, 3, -4)$ and parallel to the vector $(3, 5, -6)$ (5 marks)

QUESTION FIVE (20 MARKS)

- a) Solve by crammer's Rule

$$x - 2y + 3z = 10$$

$$3x - 6y + z = 22$$

$$-2x + 5y - 2z = -18$$

(5 marks)

- b) Find the inverse of the following matrix

$$\begin{bmatrix} 2 & 3 & 1 \\ 2 & 3 & 2 \\ 1 & 3 & 1 \end{bmatrix}$$

(5 marks)

- c) Solve the following system of equation by elimination method

$$x_1 + x_2 + x_3 = 5$$

$$x_1 + 2x_2 + 3x_3 = 10$$

$$2x_1 + x_2 + x_3 = 6$$

(5 marks)

- d) Determine the rank of the following matrix.

$$\begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 1 & 0 \end{pmatrix}$$

(5 marks)