MACHAKOS UNIVERSITY COLLEGE

(A Constituent College of Kenyatta University)
University Examinations for 2015/2016 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

…….. SEMESTER EXAMINATION FOR DEGREE IN BACHELOR OF SCIENCE IN …….

SMA 300: ORDINARY DIFFERENTIAL EQUATIONS 1

DATE: TIME:

INSTRUCTION TO CANDIDATES

Answer Question One And Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Consider the following differential equation

$$\frac{d^3y}{dx^3}+\sin x\frac{d^2y}{dx^2}+4e^x y = xe^{2x}+\cos x.$$

i) Determine the order and degree of the differential equation. (2 marks)

ii) Classify the differential equation as

(a) ordinary or partial (1mark)
(b) linear or non linear (1mark)
(c) homogenous or non-homogeneous (1mark)

b) Find the particular solution to the initial value problem $\frac{dy}{dx}=\frac{y\cos x}{1+2y}$, given that $y=1$ when $x=0$ (4 marks)

c) Find an integrating factor of the differential equation $6xydx+(4y+9x^2)dy=0$ (4 marks)

d) Classify the following first order differential equations as either linear, exact, homogeneous or seperable:

i) $e^x dx - ydy = 0$ (1 mark)

ii) $dy = \frac{-2xy}{1+x^2}dx$ (1 mark)

iii) $y' + y = \cos x$ (1 mark)

iv) $t\frac{dy}{dt} - 5t = -y$ (1 mark)

e) Determine whether $y = Ae^{-2x} + Bxe^{-2x}$ is a solution to the differential equation $y'' + 4y' + 4y = 0$ (4 marks)

f) Solve the differential equation $x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 12y = 0$ (5 marks)

g) Form a second order differential equation for $y = C_1x^2 + C_2x$ by eliminating the arbitrary constants C_1 and C_2. (4 marks)

QUESTION TWO (20 MARKS)

a) Solve the differential equation $x^2\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ (5 marks)

b) Verify that the function $\mu(x, y) = \frac{1}{x^3}$, $x > 0$ is an integrating factor for the differential equation $-2ydx + (x + x^3\sin 2y)dy = 0$. Hence solve the differential equation. (5marks)

c) For what values of k, if any, is the function $y = e^{kx}$ a solution to the differential equation $y''' + 3y'' + 2y' = 0$? (5 marks)

d) Separate the variables and hence solve the differential equation $(1 + \sin^2 x)y' + (1 + y^2)\sin 2x = 0,\ y(\frac{\pi}{2}) = 0$ (5 marks)

QUESTION THREE (20 MARKS)

a) Using the method of undetermined coefficients, solve the differential equation $\frac{d^2y}{dx^2} + 4y = 2 - 4x^2$ (6 marks)

b) Using inverse operator method or otherwise, solve the differential equation $(x^2 - 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$ about $x = 0$ (9 marks)

c) Given the equation $\frac{dy}{dx} = y^2x^2$, classify this type of first order differential equation. Hence solve it. (5 marks)

QUESTION FOUR (20 MARKS)

a) Find the power series solution of the equation $y'+3xy=1$ about . (5 marks)

b) Show that the following differential equation is homogenous $xydy-(x^2+y^2)dx=0$ .Hence solve it. (5 marks)

c) Solve the differential equation $y(1+xy)dx+x(1-xy)dy=x$ (5 marks)

d) Find the appropriate integrating factor hence solve the equation $y^2dx+xydy=0$ (5marks)

QUESTION FIVE (20 MARKS)

a) Apply the method of variation of parameters to solve the equation $(1-x)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=1-x^2$ (9 marks)

b) Using the method of separation of variables, solve the equation $y\sqrt{1-x^2}dy+x\sqrt{1-y^2}dx=0$ (5 marks)

c) The rate at which the volume of sales Q for a new type of printer increases after an advertising campaign is given by the equation $\frac{dQ}{dt}=0.04(700-Q)$, given that $Q=0$ at $t=0$. Q is the number the number of printers sold, t is time in years. Solve the differential equation to obtain the expression for Q in terms of t. Sketch the solution curve hence describe how the volume of sales should increase in time. (6 marks)