



MACHAKOS UNIVERSITY

ISO 9001:2008 CERTIFIED

**SCHOOL OF PURE AND APPLIED SCIENCES FOR THE
BACHELOR'S DEGREE**

DEPARTMENT OF MATHEMATICS AND STATISTICS.

UNIVERSITY SUPPLEMENTARY EXAMINATIONS

SMA 433: PARTIAL DIFFERENTIAL EQUATIONS II

Time 2hours

Attempt question *one (compulsory)* and any other *two questions*.

Question one (30marks)

- a.) Define a second order linear partial differential equation (4marks)
- b.) State the short form a second order partial differential equation (3marks)
- c.) State the three classes of the second order partial differential equation (4marks)
- d.) Consider the equation $2u_{xx} + 3u_{xy} + u_{yy} = 0$.
 - i.) Classify the equation (4marks)
 - ii.) Determine its characteristic equation (5marks)
 - iii.) Transform the equation to canonical form (7marks)
 - iv.) Solve the equation (7marks)

Question Two (20marks)

Prove the general equation $A \frac{\partial^2 u}{\partial x^2} + B \frac{\partial^2 u}{\partial x \partial y} + C \frac{\partial^2 u}{\partial y^2} + D \frac{\partial u}{\partial x} + E \frac{\partial u}{\partial y} + Fu = 0$

Where A, B, C, D, E, F are real constants such that $A \neq 0$.

Question Three (20marks)

Solve the wave equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$

Given that $u(x, 0) = f(x)$ and $u_t(x, 0) = g(x)$.

Question four (20marks)

Determine the characteristic of the Tricomi equation $y u_{xx} + u_{yy} = 0$ in the lower-half plane $y < 0$. Put this equation into canonical form in the upper half- plane $y > 0$.

Question five (20marks)

- a.) An infinitely long string having one end at $x = 0$ is initially at rest on the x -axis. At $t = 0$; the end $x = 0$ begins to move along the u -axis in a manner described by $u(0, t) = a \cos \sigma t$. Determine the displacement $u(x, t)$ of the string at any point at any time. (10marks)
- b.) Using Mong's method obtain the integral of $q^2 r - 2 p q s + p^2 t = 0$ in the form $y + x f(z) = F(z)$ (10marks)