MACHAKOS UNIVERSITY
ISO 9001:2008 Certified

UNIVERSITY EXAMINATIONS 2016/2017

THIRTH YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE MATHEMATICS, MATHEMATICS & COMPUTER SCIENCE & EDUCATION

SMA 301: REAL ANALYSIS

NSTRUCTIONS TO CANDIDATES

(a) Answer ALL the questions in Section A and ANY TWO Questions in Section B

SECTION A

QUESTION ONE 30 Marks (Compulsory)

a) Define the following terms
   i) Absolute convergence
   ii) Lower darboux sum
   iii) Upper darboux
   iv) Conditional convergence
   v) Radius of convergence (5 marks)

b) Find the radius of convergence of the following series $\sum \frac{1}{n^2} x^n$ in $\mathbb{R}$. (5 marks)

c) Prove that if (X, ρ) is a metric space and $f: (X, \rho) \to (\mathbb{R}, \partial)$ is continuous and if $a, b \in X$ where $f(a) > 0$ and $f(b) < 0$. Then there are neighbourhoods $N(a),\ N(b)$ of a, b respectively such that $f(x) > 0\ \forall\ x \in N(a)$ and $f(x) < 0\ \ \forall\ \ x \in N(b)$. (5 marks)

d) Let $x_n\ and\ y_n$ be convergent sequence of elements of $\mathbb{C}$ with limits x and y respectively. Prove that $\lim\limits_{n \to \infty} (x_n + y_n) \quad = x + y$ (5 marks)

e) Find the sum of the series
$$\sum_{n=0}^{\infty} 2 \sin\left(\frac{\pi}{6}\right)^n$$ (5 marks)

f) Prove that all the monotonic functions on bounded intervals are functions of bounded variations. (5 marks)

QUESTION TWO 20 MARKS

a) Test the convergence or divergence of the series $\sum_{n \in N} \frac{1}{n^p}$ if

i) $p > 1$

ii) $p < 1$

iii) $p = 1$ (4 marks each)

b) Prove that if $\sum z_n$ is absolutely convergent then $\sum z_n$ is convergent the converse is not true. (5 marks)

c) Determine whether the following series diverges or converges
$\sum_{n=1}^{\infty} \frac{2n}{5n+4}$ (3 marks)

QUESTION THREE 20 MARKS

a) Let $f: \mathbb{R} \to \mathbb{R}$ defined by $(x) = \begin{cases} 1 \; if \; x \in \mathbb{R} \\ 0 \; if \; x \in \mathbb{R} \end{cases}$. Does the right and the left sided limit exist?

(6 marks)

b) Let $f(x) = \begin{cases} x + 4, & if \; 0 < x < 1 \\ x + 5 & if \;\; x \geq 1 \end{cases}$. Is the function continuous? (6 marks)

c) If V is monotonic increasing on $[a, b]$ that is $x, y \epsilon \; [a, b]$ and $< y$, then prove that $v(x) \leq v(y)$. (7 marks)

QUESTION FOUR 20 MARKS

a) Prove that if $f \epsilon Bv[a, b]$ and $D = v - f$ then D is monotonic increasing on $[a, b]$.

(7 marks)

b) Prove that if $f: [a, b] \to R$ be bounded and α increasing on $[a, b]$ if $p_1 \; and \; p_2$ are any 2 partitions of $[a, b]$ then $l(f, \alpha, p_1) \; \leq U(f, \alpha, p_2)$. (7 marks)

c) Prove that if $f \epsilon Bv[a, b]$ if and only if it can be expressed as the difference of two monotonic increasing functions on $[a, b]$. (6 marks)

QUESTION FIVE 20 MARKS

a) Evaluate $\int_0^4 x d([x] - x)$ where $[x]$ stands for the greatest integer less than x . (5 marks)

b) Let $f(x) = \begin{cases} 2x + 1, & if \; x \in (0,1) \\ -4 & if \;\; x \in (1,2) \end{cases}$

And $\alpha(x) = \begin{cases} x+3, & if\ x \in (0,1) \\ 6-x & if\ \ x \in (1,2) \end{cases}$ is $f \in \mathcal{R}(\alpha)$ On (0,2). (5 marks)

c) Define f by f(x)= 1 ∀ $x\epsilon[a,b]$ show that $f\epsilon R[a,b]$ such that $\int_a^b f(x)dx = b-a$.

(5 marks)

d) Consider the series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$,determine whether the series converges or diverges using the integral test. (5 marks)