MACHAKOS UNIVERSITY

University Examinations 2017/2018

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

AUGUST SESSION EXAMINATIONS FOR BACHELOR OF EDUCATION

SMA 330: NUMERICAL ANALYSIS 1

DATE: SCHOOL BASED **TIME:**

INSTRUCTIONS

Attempt question **one** and any other **two**

QUESTION ONE (30 MARKS)

a) i.) Given the number 763.91. Calculate the maximum error in this number's approximation. (3 marks)

ii.) Evaluate as accurate as possible $\frac{3.45}{4.87}$ (4 marks)

iii.) obtain a second degree polynomial approximation to $f(x)=(1+x)^{\frac{1}{2}}$, $x\in[0,0.1]$ using the Taylors series expansion about $x=0$. Use the expansion to approximate $f(0.05)$ and find a bound of the truncation error. (6 marks)

b) i.) construct a finite difference table for $f(x)=e^{x}$; $0.2(0.01)0.28$ (5 marks)

ii.) show that $\delta=\nabla(1-\nabla)^{\frac{-1}{2}}$ (2 marks)

c) Evaluate $\int_{0}^{1}x^{3}dx$ with five sub-intervals by trapezoidal rule. (5 marks)

d) Determine the positive roots of $x^{4}-x-10=0$ by five iterations. (5 marks)

QUESTION TWO (20 MARKS)

a) i.) state Lagrange's interpolation formula (1 mark)

ii.) using unequal intervals in an experiment a student got the following values of a function $f(x)$;

x	1	2	-4
$f(x)$	3	-5	4

Represent the function $f(x)$ approximately by a polynomial of degree 2.

(5 marks)

b) Find a real root of the equation $xe^{x} - \cos x = 0$ using Newton- Raphson method to 3 s.f.

(9 marks)

c) A rocket is launched from the ground. Its acceleration measured every 5 seconds is tabulated below. Using Simpson's rule. Find the velocity and the position of the rocket at $t = 40\,\text{sec}\,onds$ (5 marks)

t	0	5	10	15	20	25	30	35	40
$a\ (t)$	$40\cdot0$	$42\cdot25$	$48\cdot50$	$51\cdot25$	$54\cdot35$	$59\cdot48$	$61\cdot50$	$64\cdot30$	$68\cdot70$

QUESTION THREE (20MKS)

a) calculate $\int_{0}^{\frac{1}{2}} \frac{x}{\sin x}\,dx$ using Romberg integration with step size $h = \frac{1}{16}$ (10 marks)

b) using the method of undetermined coefficients find the nodes and weights of the

Quadrature formula $\int_{0}^{\infty} e^{-x} f(x)dx = \lambda_0 f(x_0) + \lambda_1 f(x_1)$ (10 marks)

QUESTION FOUR (20 MARKS)

a) Consider the table below;

x	1.72	1.73	1.74	1.75	1.76
$y = f(x)$	0.17907	0.17728	0.17552	0.17377	0.17204

Find;

i) $f'(1.72)$ (2 marks)

ii) $f'(1.76)$ (6 marks)

b) Find the first and the second derivatives of the function tabulated below at $x = 0.6$

x	0.4	0.5	0.6	0.7	0.8
y	1.5836	1.7974	2.0442	2.3275	2.6511

(10 marks)

QUESTION FIVE (20 MARKS)

a) Given $f(2) = 4$; $f(2.5) = 5.5$ find the linear interpolating polynomial using;

i) Aitken's iterated interpolation. (4 marks)

ii) Newton's divided difference interpolation (3 marks)

iii) Hence find an approximate value of $f(2.2)$ (3 marks)

b) i.) construct the difference table for the sequence of values $f(x) = [0,0,0,\varepsilon,0,0,0]$ where ε is an error. (6 marks)

ii) Show that the error spreads and increases in magnitude as the order of differences is increased. (2 marks)

iii) Show that the errors in each column have binomial coefficients. (2 marks)