MACHAKOS UNIVERSITY

University Examinations for 2020/2021 Academic Year

SCHOOL OF PHYSICAL AND BIOLOGICAL SCIENCES

DEPARTMENT OF MATHEMATICS

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

SST465: MEASURE AND PROBABILITY

DATE: 20/8/2021 **TIME: 8.30-10.30 AM**

INSTRUCTIONS

Answer question ONE and any other TWO questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Describe Probability theory (2 marks)

b) Describe the term independence as used in probability theory. (2 marks)

c) Consider the coin experiment of tossing a coin n times and recording the score (1 for heads or 0 for tails) for each toss.

  i) Identify a parameter of the experiment.

  ii) Interpret the experiment as n Bernoulli trials. (4 marks)

d) Consider an experiment with S as the set of outcomes. The collection F of events is required to be a σ-algebra of subsets of S. Describe the conditions to be fulfilled in this case. (4 marks)

e) State the following laws of algebra of sets (6 marks)

  a) Identity

  b) Complement

  c) Involution

f) Describe a random variable using an hypothetical example of an experiment. (2 marks)

g) A probability measure (or probability distribution) P on the sample space (S,F) is a real-valued function defined on S. Outline the axioms that should be satisfied. (3 marks)

h) Let A and B be events with P(B)>0. Define the conditional probability of A given B (2 marks)

i) Describe the law of large numbers (2 marks)

j) A *probability space* (S,F,P), consists of three essential parts. Describe the three parts. (3 marks)

QUESTION TWO (20 MARKS)

a) Prove the Prove Idempotent Law of algebra of sets (4 marks)

b) Prove the Associative Laws of algebra of sets (4 marks)

c) Prove the commutative law of algebra of sets. (4 marks)

d) Prove the distributive law of algebra of sets. (4 marks)

e) Prove the demorgans law of algebra of sets (4 marks)

QUESTION THREE (20 MARKS)

a) Suppose that X is a random variable taking values in T, and that A,B∈F. Proof that:-

  i) $\{X\in A\cup B\}=\{X\in A\}\cup\{X\in B\}$

  ii) $\{X\in A\cap B\}=\{X\in A\}\cap\{X\in B\}$

  iii) $\{X\in A\setminus B\}=\{X\in A\}\setminus\{X\in B\}$

  iv) $A\subseteq B\Rightarrow\{X\in A\}\subseteq\{X\in B\}$

  v) If A and B are disjoint, then so are $\{X\in A\}$ and $\{X\in B\}$. (5 marks)

b) Consider the coin experiment with n=4, and Let Y denote the number of heads.

  i) Give the set of outcomes S in list form.

  ii) Give the event $\{Y=k\}$ in list form for each $k\in\{0,1,2,3,4\}$. (6 marks)

  iii) Consider the dice experiment with n=2 standard dice. Let S denote the set of outcomes, A the event that the first die score is 1, and B the event that the sum of the scores is 7. Give each of the following events in the form indicated (6 marks)

    a. S in Cartesian product form

    b. A in list form

    c. B in list form

    d. $A\cup B$ in list form

    e. $A\cap B$ in list form

    f. $A^c\cap B^c$ in predicate form

  (iv) Describe the Bayes theorem (3 marks)

QUESTION FOUR (20 MKS)

a) Describe using examples how can we construct probability measures? (5 marks)

b) Prove that if μ is a positive measure on S with $0<\mu(S)<\infty$ then P defined below is a probability measure on (S,F)

$P(A)=\mu(A)/\mu(S), A\in F$ (5 marks)

c) Describe with an example the discrete and continuous probability distribution in the sample space *(S,F)* (5 marks)

d) Suppose that A and B are events in an experiment with $P(A)=1/3$, $P(B)=1/4$, $P(A\cap B)=1/10$. Express each of the following events in the language of the experiment and find its probability. (5 marks)

i) $A\backslash B$

ii) $A\cup B$

iii) $A^c\cup B^c$

iv) $A^c\cap B^c$

v) $A\cup B^c$

QUESTION FIVE (20 MARKS)

a) In a certain population, 30% of the persons smoke cigarettes and 8% have COPD (Chronic Obstructive Pulmonary Disease). Moreover, 12% of the persons who smoke have COPD.

i) What percentage of the population smoke and have COPD?

ii) What percentage of the population with COPD also smoke?

iii) Are smoking and COPD positively correlated, negatively correlated, or independent? (6 marks)

b) A company has 200 employees: 120 are women and 80 are men. Of the 120 female employees, 30 are classified as managers, while 20 of the 80 male employees are managers. Suppose that an employee is chosen at random. (6 marks)

i) Find the probability that the employee is female.

ii) Find the probability that the employee is a manager.

iii) Find the conditional probability that the employee is a manager given that the employee is female.

iv) Find the conditional probability that the employee is female given that the employee is a manager.

iv) Are the events female and manager positively correlated, negatively correlated, or independent?

c) Suppose that we have an infinite sequence of coins labeled 1,2,… Moreover, coin n has probability of heads 1/na for each n∈N+, where a>0 is a parameter. We toss each coin in sequence one time. In terms of a, find the probability of the following events: (4 marks)

i) infinitely many heads occur

ii) infinitely many tails occur

d) Consider the experiment of rolling two standard, fair dice and recording the sequence of scores (X,Y). Prove that X and Y are independent and have the same distribution, but are not equivalent. (4 marks)