MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATIONS FOR

BACHELOR OF SCIENCE (MATHEMATICS AND STATISTICS)

BACHELOR OF SCIENCE (COMPUTER SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

SMA 305 – COMPLEX ANALYSIS I.

DATE: **TIME:**

INSTRUCTIONS: *Answer Question 1 And Any Other 2 Questions.*

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Find all the 4th roots of unity (1) and plot these values in the argand diagram (5 marks)

b) Discuss continuity and uniform continuity of the function $f(z) = z^2$ in the region $|z| < 5$ (5 marks)

c) Let $f = \frac{az+b}{cz+d}$ (where a, b, c, d are constants) be a mobius function, find f^{-1} (5 marks)

d) When is a complex valued function said to be analytic? Is the function $f(z) = |z|^2$ analytic in the complex plane? (5 marks)

e) Investigate for convergence of the series $\sum_{n=1}^{\infty} \frac{\left(\frac{1}{2}z+1-i\right)^n}{3^n}$ using the ratio test (5 marks)

f) Evaluate $\int_c \bar{z}\, dz$ from $z = 0$ to $z = 4 + 2i$ along the curve $z = t^2 + it$ (5 marks)

QUESTION TWO (20 MARKS)

a) Find all complex numbers for which $cosz = 3.5$ (6 marks)

b) Determine $u(x, y)$ and $v(x, y)$ for the function $w = (-3)^i$ (5 marks)

c) Find all values of $\sin^{-1}(2i)$ (5 marks)

d) Express the polar form of $\left[\frac{2+i2\sqrt{3}}{1+i}\right]^2$ (4 marks)

QUESTION THREE (20 MARKS)

a) Evaluate $\oint_C \frac{1}{z^3+1} dz$ where C is a circle $|z| = 1$ using Cauchy's residue theorem (10 marks)

b) Verify Green's theorem in the plane for $\oint_c \{(2yx - x^2)dx + (x + y^2)dy\}$ where c is the closed curve of the region bounded by $y = x^2$ and $y = x$ (10 marks)

QUESTION FOUR (20 MARKS)

a) Expand $f(z) = sinz$ about $z_0 = \frac{\pi}{4}$ by the Taylor series and determine the region of convergence (7 marks)

b) Expand $f(z) = \frac{e^z}{z^2}$ by Laurent series (6 marks)

c) Let $f(z) = \frac{z-1}{z^2+9}$, compute the poles and their residue. Hence solve $\int_c f(z)dz$, where c encloses the poles (7 marks)

QUESTION FIVE (20 MARKS)

a) Show that $U = e^{-2xy} sin(x^2 - y^2)$ is harmonic. (7 marks)

b) Derive the Cauchy-Riemann equations. Hence determine the function whose real part is $x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$. (9 marks)

c) Find the image of the line $x = 1$ under the mapping $w = \frac{1}{z}$. (4 marks)