



MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS)

SMA 104: CALCULUS

DATE: 26/3/2021

TIME: 8.30-10.30 AM

INSTRUCTIONS TO CANDIDATES

Attempt question *one (compulsory)* and any other *two questions*.

QUESTION 1(30 MARKS)

- a) Consider the function $f(x) = \log_e(3x^2 - 4x + 5)$. Express the function as the composite of two functions (2 marks)

- b) Consider the function

$$f(x) = \begin{cases} 7x - 2 & x \geq 2 \\ 3x + 5 & x < 2 \end{cases}$$

Determine the one-sided limits

- i) $\lim_{x \rightarrow 2^+} f(x)$ (2 marks)
- ii) $\lim_{x \rightarrow 2^-} f(x)$ (2 marks)
- c) The function $f(x) : R \rightarrow R$ is defined by $f(x) = \frac{e^x - e^{-x}}{2}$. Calculate the inverse of $f(x)$ (3 marks)
- d) Calculate $\lim_{x \rightarrow 4^-} \frac{3}{x-4}$ (3 marks)

- e) Prove that the function $f(x) = \sin^2 x$ is continuous for every value of x on R (4 marks)
- f) A ladder $20m$ long leans against a vertical building. If the bottom of the Ladder slides away from the building horizontally at a rate of $2m/s$, how fast is the ladder sliding down the building when the top of the ladder is $12m$ above the ground. (4 marks)
- g) Use the first principle to evaluate $\frac{dy}{dx}$ for $y = \log_b x$ (5 marks)
- h) Calculate $\frac{d^2 y}{dx^2}$ for the parametric equations $x = \cos 2t$, $y = 2 \sin 2t$ (5 marks)

QUESTION TWO (20 MARKS)

Consider the function $f(x) = x^4 - 2x^2$

- a) Determine using the second derivative
- Local maximum point (3 marks)
 - Local minimum point (3 marks)
- b) Work out the coordinate of the points of inflection of $f(x)$ (4 marks)
- c) Discuss the concavity of $f(x)$ (5 marks)
- d) Sketch the graph of $f(x)$ (5 marks)

QUESTION THREE (20 MARKS)

Determine $\frac{dy}{dx}$

- a) $5y^2 + \sin y = x^2$ (4 marks)
- b) $y = \ln\left(\frac{x^2 \sin x}{\sqrt{1-x}}\right)$ (4 marks)
- c) $ye^x = \sinh(xy)$ (4 marks)
- d) $y = \tan^{-1}(2t+1)$, $x = e^{-t^2}$ (4 marks)
- e) $y = 3^{\cos x}(x^x)$ (4 marks)

QUESTION FOUR (20 MARKS)

- a) A projectile is fired straight upwards with a velocity of 40m/s ; its distance above the ground $t\text{sec}$ after being fired is given by $S(t) = -16t^2 + 40t$. $S(t)$ is the distance of the particle from the ground after it has been fired.
- Work out the acceleration at any time (t) (2 marks)
 - Determine the maximum height achieved by the projectile (4 marks)
 - Calculate the time and the velocity at which the projectile hits the ground (5 marks)
- b) A vessel containing water is in the form of an inverted cone with semi-vertical angle of 30° . there is a small hole at the vertex of the cone and the water is running out at a rate of 3cm^3 per second. Calculate the rate at which the surface area in contact with water is changing when there are $81\pi\text{cm}^3$ of water remaining in the cone. (9 marks)

QUESTION FIVE (20 MARKS)

- a) Determine;
- $\lim_{x \rightarrow 1} \frac{x^4 + 3x^3 - 13x^2 - 27x + 36}{x^2 + 3x - 4}$ (5 marks)
 - $\lim_{x \rightarrow -\infty} \frac{4x - 1}{\sqrt{x^2 + 2}}$ (5 marks)
 - $\lim_{x \rightarrow 0} \frac{|x|}{x}$ (5 marks)
- b) Given that $f(x) = \tan^{-1} x$. Calculate $f'(x)$ (5 marks)