MACHAKOS UNIVERSITY

University Examinations 2017/2018

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING

BACHELOR OF SCIENCE IN MATHEMATICS

BACHELOR OF EDUCATION

SMA102: BASIC MATHEMATICS

DATE: 5/12/2017 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS: Answer questions ONE and any other TWO questions.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Solve $2x^2 - 5x - 3 = 0$ by use of factorization method. (3 marks)

b) Divide $3x^3 + 2x^2 - x - 1$ by $x - 1$ (3 marks)

c) Without using a calculator evaluate $\frac{9!}{2!7!}$ (2 marks)

d) Use Pascal's triangle to expand $\left(10 + \frac{2}{x}\right)^5$ (3 marks)

e) Let A, B and C be sets. Prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$ (5 marks)

f) Construct truth tables for the following compound propositions;

i) $\sim P \vee Q$ (3 marks)

ii) $\sim P \Leftrightarrow \sim Q$ (3 marks)

g) Use the principle of mathematical induction to prove that for any positive integer n

$$1 + 2 + 3 + 4 \ldots . + n = \frac{n(n+1)}{2}$$ (4 marks)

h) Use remainder theorem to find the remainder when $x^5 - 4x^3 + 2x + 3$ is divided by $x + 2$ (2 marks)

QUESTION TWO (20 MARKS)

a) Prove that $(A \cap B)^C = A^C \cup B^C$ by the Elements Argument method. (4marks)

b) In a class of 50 college freshmen, 30 are studying C^{++}, 25 are studying java and 10 are studying both languages. With the help of a well labeled Venn diagram find the number of freshmen who are studying either C^{++} or java language? (4 marks)

c) Consider the following propositions; P: mathematicians are generous, Q: first years hate algebra. Write the compound propositions symbolized by;

i. $PV \sim Q$ (1 mark)

ii. $\sim (P \wedge Q)$ (1 mark)

iii. $\sim P \Rightarrow \sim Q$ (1 mark)

iv. $\sim P \Leftrightarrow \sim Q$ (1 mark)

d) Write $40 \times 39 \times 38 \times 37$ in factorial notation (3 marks)

e) A committee of six is to be formed from 9 women and three men. In how many ways can the members be chosen so as to include at least one man? (5 marks)

QUESTION THREE (20 MARKS)

a) Express $\alpha^3 + \beta^3$ in terms of $\alpha + \beta$ and $\alpha\beta$ (3 marks)

b) Given that $\frac{x+iy}{2+i} = 5 - i$ solve for x and y. (4 marks)

c) Use De Moivere's theorem to show that $\frac{\sin 5\theta}{\sin\theta} = 16(\cos\theta)^4 - 12(\cos\theta)^2 + 1$ (8 marks)

d) Obtain the first four terms of the expression $\left(1+\frac{1}{2}x\right)^{10}$ in ascending powers of x and hence estimate the value of $(1.005)^{10}$ correct to four decimal places. (5 marks)

QUESTION FOUR (20 MARKS)

a) In a lottery, a total of a thousand tickets were sold. Determine the winners of the first, second and third prizes if three tickets are drawn one at a time. (3 marks)

b) A girl wants to invite eight friends to her birthday party but there is only room for four of them. In how many ways can she choose whom to invite if two of them are sisters and must not be separated. (5 marks)

c) Let $A = \{0,1,2\}$ and $B = \{a, b\}$ find $A \times B$ (3 marks)

d) Prove that $A \cup (B - A) = A \cup B$ (5 marks)

e) How many even numbers greater than 2000 can be formed from the digits 1,2,4,8 if a digit can only be used once in each number. (4 marks)

QUESTION FIVE (20 MARKS)

a) Find all the values of z for which $z^5 + 32 = 0$ (7marks)

b) Use the principle of mathematical induction to prove that for any natural number $n \geq 1$

$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$ (5marks)

c) Find the number of permutations of the letters of the word. MISSISSIPPI (3marks)

d) Expand and simplify $\left(a+\frac{1}{2}\right)^4 + \left(a-\frac{1}{2}\right)^4$ (5 marks)