



MACHAKOS UNIVERSITY

UNIVERSITY EXAMINATIONS 2017/2018
UNIVERSITY EXAMINATION FOR THE DEGREE OF BACHELOR
OF SCIENCE STATISTICS AND PROGRAMMING
SST 103: LINEAR ALGEBRA

DATE:..... TIME:.....

INSTRUCTIONS: Answer QUESTION ONE and any other TWO QUESTIONS

QUESTION ONE 30 Marks (Compulsory)

- a) Calculate the cross product of the vectors $\vec{u} = (1, 2, 3)$ and $\vec{v} = (-1, 1, 2)$. (3 marks)
- b) Find the angle between $u = 2i + 2j + 2k$ and $v = i + j + k$ (3 marks)
- c) Find the inverse of the following matrix (4 marks)
- $$\begin{bmatrix} 0 & 2 & 4 \\ 2 & 4 & 2 \\ 3 & 3 & 1 \end{bmatrix}$$
- d) Solve the linear system (5 marks)
- $$\begin{aligned} x + 2y + 3z &= 4 \\ 3x + 8y + 7z &= 20 \\ 2x + 7y + 9z &= 23 \end{aligned}$$
- e) Find the equation of the plane through the point $(-1, 2, 3)$ and perpendicular to the plane $2x - 3y + 4z = 1$ and $3x - 5y + 2z = 3$ (5 marks)
- f) Determine c so that $a = ci + 5j - 3k$ and $b = 3i + 4j - 2k$ are orthogonal (5 marks)
- g) Find the parametric and symmetric equation of the line passing through the point $(2, -3, 4)$ and parallel to the vector $(3, 5, -6)$ (5 marks)

Question two 20 marks

- a) Show that $W = \{(x, y) / x = 2y\}$ is a subspace for $\mathbb{R}^2$. (4 marks)
- b) Prove that the diagonals of a rhombus are perpendicular. (5 marks)

c) Find the parametric and the symmetric equations of the line passing through the point $(2, 3, -4)$ and parallel to the vector $(3, 5, -6)$ (5 marks)

d) Define a vector space (6 marks)

Question Three 20 marks

a) Find the determinant of the following matrix

$$\begin{bmatrix} 2 & 5 & 0 \\ 1 & 4 & 8 \\ 4 & 7 & 1 \end{bmatrix}$$

(3 marks)

b) Find the ranks of the following matrix $\begin{bmatrix} 1 & 3 & 1 \\ 2 & 3 & 2 \\ 1 & 3 & 0 \end{bmatrix}$

(5 marks)

c) Find the minors and cofactors of the following matrix $\begin{bmatrix} 2 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 3 & 1 \end{bmatrix}$

(6 marks)

d) Transpose the following matrix $\begin{bmatrix} 2 & 1 & 3 \\ 2 & 3 & 7 \\ 1 & 4 & 3 \end{bmatrix}$

(2 marks)

e) Reduce the following matrix into echelon form $\begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 4 \\ 1 & 6 & 1 \end{bmatrix}$

(5 marks)

Question four 20 marks

a) Solve the following system of equation by elimination method

$$x_1 + x_2 + x_3 = 5$$

$$x_1 + 2x_2 + 3x_3 = 10$$

$$2x_1 + x_2 + x_3 = 6$$

(4 marks)

b) Solve the simultaneous equation using the gauss-Elimination method

$$2x_1 - 4x_2 + 6x_3 = 20$$

$$6x_1 - 12x_2 + 2x_3 = 44$$

$$-4x_1 + 10x_2 - 4x_3 = -36$$

(5 marks)

c) Solve by Crammer's Rule

$$x - 2y + 3z = 10$$

$$3x - 6y + z = 22$$

$$-2x + 5y - 2z = -18 \quad (5 \text{ marks})$$

d) Determine the value of 'a' so that the following systems in unknown x , y and z has

- i). No solution
- ii). More than one solution
- iii). Unique solution

$$x - 3z = -3$$

$$2x + ay - z = -2$$

$$x + 2y + az = 1 \quad (6 \text{ marks})$$

Question five 20 marks

- a) Find the equation of the plane passing through the point $(3, -1, 7)$ and perpendicular to the vector $n = (4, 2, -5)$. (5 marks)
- b) Find the distance D between the point $(1, -4, -3)$ and the plane $2x - 3y + 6z = -1$ (5 marks)
- c) Find the parametric equations for the line of intersection of the plane $3x + 2y - 4z - 6 = 0$ and $x - 3y - 2z - 4 = 0$ (5 marks)
- d) Show that the vectors $u=(1,-1,0)$ $v=(1,3,-1)$ and $w=(5,3,-2)$ are linearly dependent (5 marks)