MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR …… SEMESTER EXAMINATION FOR

BACHELOR OF EDUCATION

SMA 332: METHODS OF APPLIED MATHEMATICS

DATE: **TIME:**

INSTRUCTION:

Answer Question One and Any Other Two Questions

QUESTION ONE (30 MARKS) (COMPULSORY)

a) Decompose the equation $u_{tt} = c^2 u_{xx}$, $0 < x < L, t > 0$ into ordinary differential equations (ODEs) in variables X and T. (6 marks)

b) Sate the Fourier series expansion of a function $f(x)$ over the interval $-\pi < x < \pi$. (2 marks)

c) Show that the Fourier coefficient a_n of the Fourier series, you have defined in b) above is given by; $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\, dx, \quad n = 1,2,3,\ldots$ (5 marks)

d) Determine the Laplace transform of $f(x) = \cos ax$. (5 marks)

e) Use the Convolution theorem to determine the inverse Laplace transform of the quotient

$$\frac{6}{(s^2+1)(s^2+9)}$$

(6 marks)

f) Show that the Laplace transform $\frac{\partial^2 u}{\partial t^2}$ is given by $s^2 U(x,s) - su(x,0) - u_t(x,0)$ (6 marks)

QUESTION TWO (20 MARKS)

a) Classify the 2nd order partial differential equation (PDE) $y^2 u_{xx} + x^2 u_{yy} = 0$ and transform it to canonical form. (13 marks)

b) Obtain the Fourier series for the function

$$f(x) = 2x + 1, \quad -\pi < x < \pi$$
$$f(x) = f(x + 2\pi)$$
(7 marks)

QUESTION THREE (20 MARKS)

a) Use partial fractions to determine the inverse Laplace transform of the quotient:

$\frac{s^2}{(s+3)^3}$ (8 marks)

b) Solve the following 2nd differential equation using Laplace transform method;

$y'' + 5y' + 6y = 2e^{-t}$ subject to $y(0) = 1,\ y'(0) = 0$ (12 marks)

QUESTION FOUR (20 MARKS)

a) A heavy-duty wire of length a that is fixed at $x = 0$ and free to move at the other end $x = a$ is initially at rest and subjected to a constant force. Assuming that the wire's displacement $u(x,t)$ happens only in the x direction and $u_x(a,t) = D$ where D is the displacement per unit length.

i) Show that the Laplace transform of the associated wave equation is given by the ODE $\frac{d^2U(x,s)}{dx^2} = \frac{s^2}{c^2}U(x,s)$ (5 marks)

ii) Show that the solution to the ODE given in 4 a) i) above is

$U(x,s) = \frac{cD\sinh(sx/c)}{s^2\cosh(sa/c)}$ (10 marks)

b) Show that $\int_{-1}^{1} \cos m\pi x \sin n\pi x dx = 0$ where m, n are integers. (5 marks)

QUESTION FIVE (20 MARKS)

a) A thin bar of length π units is place in boiling water (temperature 100^oC). After reaching 100^oC throughout, the bar is removed from the boiling water. With the lateral sides kept insulated, suddenly, at time $t = 0$, the ends are immersed in a medium with constant freezing temperature 0^oC. Taking c=1, find the temperature $u(x,t)$ for $t > 0$ using the separation of variables method

(10 marks)

b) Obtain the general solution to the 2nd order PDE $yu_{tt} - k^2yu_{yy} = 2k^2u_r$ where k is a constant. (10 marks)