



MACHAKOS UNIVERSITY

University Examinations for 2020/2021

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF EDUCATION (ARTS)

BACHELOR OF EDUCATION (SCIENCE)

SMA 432: PARTIAL DIFFERENTIAL EQUATIONS I

DATE: 13/8/2021

TIME: 8:30 – 10:30 AM

INSTRUCTIONS: Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) Under what conditions can the Lagrange equation $Pp + Qq = R$ be classified as: Linear, Semi-Linear and Quasi-Linear. (4 marks)
- b) Find a partial differential equation arising from $z = ax^2 + by^2 + ab$ (4 marks)
- c) Find the complete solution to $(y - z)\frac{\partial z}{\partial x} + (x - y)\frac{\partial z}{\partial y} = z - x$ (5 marks)
- d) Find the integral curves of the equations:
- i) $\frac{dx}{x + y} = \frac{dy}{x + y} = \frac{dz}{-(x + y + 2z)}$ (6 marks)
- ii) $y^2 zp - x^2 zq = x^2 y$ (5 marks)
- iii) $\frac{dx}{xz - y} = \frac{dy}{yz - x} = \frac{dz}{1 - z^2}$ (6 marks)

QUESTION TWO (20 MARKS)

- a) Show Pfaffian differential equation given by $z(x + y^2)dx + z(z + x^2) - xy(x + y)dz = 0$ is integrable. (7 marks)
- b) Find the solution of the equation $(x^2 - y^2 - z^2)p + 2xyq = 2xz$ (9 marks)
- c) Eliminate the arbitrary constant from $z = ax^2 - by^3 + 4ab$ (4 marks)

QUESTION THREE (20 MARKS)

- a) Given that $u \equiv u(x, y, z) = c_1$, $v \equiv v(x, y, z) = c_2$ are solutions of $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$. Show that $F(u, v) = 0$ is a general solution of the Lagrange's equation. (10 marks)
- b) Find the integral curves of the equation $\frac{dx}{y(x+y)+az} = \frac{dy}{x(x+y)-az} = \frac{dz}{z(x+y)}$ (10 marks)

QUESTION FOUR (20 MARKS)

- a) Consider a surface $F(x, y, z) = 0$ and let this surface intersect with the family of surfaces $G(x, y, z) = k$. Using Cramer's rule, derive the equations:
 $P' = RF_y - QF_z, Q' = PF_z - RF_x, R' = QF_x - PF_y$ (10 marks)
- b) Find the surface which intersects the surfaces of the system $z(x+y) = c(3z+1)$ and which passes through the circle $x^2 + y^2 = 1, z = 1$ (10 marks)

QUESTION FIVE (20 MARKS)

- a) Determine the general solution to the P.D.E.
 $xp + yq = 3z$ (6 marks)
- b) Eliminate the arbitrary function from $\phi(x + y + z, x^2 + y^2 + z^2) = 0$ (4 marks)
- c) Find the integral surface of the equation $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$ which contains the straight line $x + y = 0, z = 1$ (10 marks)