



MACHAKOS UNIVERSITY

University Examinations for 2021/2022

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

SAC 101: FUNDAMENTALS OF ACTUARIAL MATHEMATICS

DATE: 26/1/2022

TIME: 8:30 – 10:30 AM

INSTRUCTION: ANSWER QUESTION ONE [COMPUSLSORY] AND TWO OTHER QUESTIONS

- a) State the distinguishing feature between simple interest and compound interest. (1 mark)
- b) Calculate the time in days for 1500 to accumulate to 1550 at
- i) Simple interest rate of 5% per annum (2 marks)
 - ii) A force of interest of 5% per annum (2 marks)
- c) The effective compounding rate of interest per annum on a certain building society account is currently 7%, but in 2 years' time it will be reduced to 6%. Find the accumulation in 5 years' time of an investment of £4,000 in this account. (3 marks)
- d) For a rate of interest of 7% per annum, convertible monthly, calculate the equivalent rate of:
- i) interest per annum convertible half yearly (2 marks)
 - ii) discount per annum convertible monthly (2 marks)
- e) Derive the following actuarial functions:
- i) $a_{n|}$ (3 marks)
 - ii) $\ddot{s}_{n|}$ (3 marks)
- f) Given that $i^{(m)} = 0.1844144$ and $d^{(m)} = 0.1802608$. Find the value of m. (4 marks)

- g) A company has borrowed Sh. 50,000,000 from a bank. The loan is to be repaid by level instalments, payable annually in arrear for ten years from the date the loan is made. The annual instalments are calculated at an effective rate of interest of 9% per annum. Calculate the total amount of interest which will be paid over the ten-year term. (4 marks)
- h) Given the table below

x	l_x
0	100,000
1	97,551
2	97,398
3	97,302

Calculate the probability

- i) Of a one-year-old surviving to his/her third birthday? (2 marks)
- ii) That a newborn will die between his/her first and third birthday? (2 marks)

QUESTION TWO (20 MARKS)

- a) The force of interest, $\delta(t)$, is

$$\delta(t) = \begin{cases} 0.04 & \text{for } 0 < t \leq 5 \\ 0.01(t^2 - t) & \text{for } 5 < t \end{cases}$$

- i) Calculate the present value of a unit sum of money due at time $t = 10$. (6 marks)
- ii) Calculate the effective rate of interest over the period $t = 9$ to $t = 10$. (4 marks)
- iii) In terms of t , determine an expression for $v(t)$, the present value of a unit sum of money due during the period $0 < t \leq 5$. (2 marks)

- b) A university student receives a 3-year scholarship grant. The payments under the grant are as follows:

Year 1 £5,000 per annum paid continuously.

Year 2 £5,000 per annum paid monthly in advance.

Year 3 £5,000 per annum paid half yearly in advance.

Calculate the total present value of these payments at the beginning of the first year using a rate of interest of 8% per annum convertible quarterly. (8 marks)

QUESTION THREE (20 MARKS)

- a) The rate of interest is 4.5% per annum effective.

Calculate:

- i) the nominal rate of discount per annum convertible monthly. (2 marks)
 - ii) the nominal rate of interest per annum convertible quarterly. (2 marks)
 - iii) the effective rate of interest over a five year period. (2 marks)
- b) The force of interest $\delta(t)$ at time t is $a + bt^2$ where a and b are constants. An amount of Sh. 20,000 invested at time $t = 0$ accumulates to Sh. 21,000 at time $t = 5$ and Sh. 23,000 at time $t = 10$. Determine a and b . (8 marks)
- c) An annuity certain with payments of Sh. 15,000 at the end of each quarter is to be replaced by an annuity with the same term and present value, but with payments at the beginning of each month instead. Calculate the revised payments, assuming an annual force of interest of 10%. (6 marks)

QUESTION FOUR (20 MARKS)

- a) A loan of Ksh. 8,000,000 is repayable over 25 years by level monthly instalments in arrears of capital and interest. The repayments are calculated using an effective rate of interest of 8% per annum. Calculate:
- i) The capital repaid in the first monthly instalment. (6 marks)
 - ii) The total amount of interest paid during the last six years of the loan. (4 marks)
 - iii) The interest included in the final monthly payment. (3 marks)
- b) Sh. 25,000 is invested at a discount rate of 18% *p.a.* convertible monthly for the first 3 months, followed by an interest rate of 20% *p.a.* convertible quarterly for the next 9 months. Calculate the accumulated sum at the end of the year. (4 marks)

- c) An investment is discounted for 28 days at a simple rate of discount of 4.5% per annum. Calculate the annual effective rate of interest. (3 marks)

QUESTION FIVE (20 MARKS)

- a) The force of interest, $\delta(t)$, is a function of time and at any time t , measured in years, is given by the formula

$$\delta(t) = \begin{cases} 0.04 + 0.003t^2 & \text{for } 0 < t \leq 5 \\ 0.01 + 0.03t & \text{for } 5 < t \end{cases}$$

- (i) Calculate the amount to which Sh. 100,000 will have accumulated at $t=7$ if it is invested at $t = 3$. (6 marks)
- (ii) Calculate the constant rate of discount per annum, convertible monthly, which would lead to the same accumulation as that in (a) being obtained. (4 marks)
- b) Show that
- (i) $\frac{1}{\ddot{a}_{n|}} = \frac{1}{s_{n|}} + d$ (4 marks)
- (ii) $\ddot{a}_{n|} = 1 + a_{n-1|}$ (3 marks)
- (iii) $\ddot{s}_{n|} = (1 + i)s_{n|}$ (3 marks)