MACHAKOS UNIVERSITY

University Examinations for 2024/2025 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR DEGREE IN BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS ENGINEERING BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING BACHELOR OF SCIENCE IN CIVIL ENGINEERING

ECU 300: ENGINEERING MATHEMATICS IX

DATE: 17TH JANUARY, 2025 **TIME: 11.00 AM – 1.00 PM**

INSTRUCTIONS:
Answer question ***ONE (Compulsory)*** *and* ***any other TWO*** *questions*

QUESTION ONE (30 MARKS)

a) Given vectors $\vec{P}$ and $\vec{Q}$ and the angle between them is β, use cosine rule to prove that $Cos\beta = \frac{\vec{P}\cdot\vec{Q}}{\left\|\vec{P}\right\|\left\|\vec{Q}\right\|}$ (5 marks)

b) Prove that $\vec{a}\times\vec{b}$ is orthogonal to both $\vec{a}$ and $\vec{b}$ (5 marks)

c) Use scalar triple product to verify that the vectors $\vec{w} = i + 5j - 2k$, $\vec{x} = 3i - j$ and $\vec{y} = 5i + 9j - 4k$ are coplanar (5 marks)

d) Find the velocity vector and magnitude of the velocity of particle which moves along the trajectory $\vec{r}(t) = 2\cos(t)\hat{i} + 2\sin^2(t)\hat{j}$ (4 marks)

e) Find all the second order partial derivatives of $z = 4x^2 - 8xy^4 + 7y^5 - 3$ z (4 marks)

f) Compute $\int_0^1 \int_{\frac{y^2}{2}}^{\sqrt{y}} dxdy$ (4 marks)

g) Determine whether the vector field $\vec{F} = (x, y, 5x)$ is conservative (3 marks)

QUESTION TWO (20 MARKS)

a) Show that $\lim_{t\to 1} \vec{r}(t) = \frac{t^2-t}{t-1}\hat{i} + \sqrt{t+8}\,\hat{j} + \frac{Sin\,\pi t}{ln\,t}\hat{k}$ exists but $\vec{r}(t)$ is not continuous at $t = 1$ (5 marks)

b) Find $\frac{dz}{dt}$ if $z = ln(3x^2 + y^3), x = e^{2t}, y = t^{\frac{1}{3}}$ (7 marks)

c) Find $\frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}$ if $z = 16 - 4x^2 - y^2, x(u,v) = u\,sin\,v,\ y(u,v) = vcos\,u$ (8 marks)

QUESTION THREE (20 MARKS)

a) Find all the stationary points of $f(x,y) = xy^2 - x^2y^2 + x^4 + 3$ (10 marks)

b) Determine the volume of the region that lies behind the plane $x + y + z = 8$ and in front of the region in the yz plane bounded by $z = \frac{3}{2}\sqrt{y}$ and $z = \frac{3}{4}y$. (10 marks)

QUESTION FOUR (20 MARKS)

a) Find dS on the xy plane on the hemisphere $x^2 + y^2 + z^2 = 4;\ z \geq 0$ (5 marks)

b) Use Stoke's theorem to evaluate $\int_c (x+y)dx + (2x-z)dy + (y+z)dz$ where c is the boundary of triangle with vertices $(2,0,0), (0,3,0), (0,0,6)$ (10 marks)

c) Show that $div(Curl\,\vec{V}) = 0$ (5 marks)

QUESTION FIVE (20 MARKS)

a) Using Divergence theorem evaluate $\iint_S \vec{F} \bullet \hat{n}dS$ where $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ (10 marks)

b) Evaluate $\iint_S \vec{A} \bullet \hat{n} dS$ where $\vec{A} = 18z\hat{i} - 12\hat{j} + 3y\hat{k}$ and S is the part of the plane $2x + 3y + 6z = 12$ which is in the xy plane (10 marks)