



MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (ACTUARIAL MATHEMATICS I)

SAC 301: ACTUARIAL MATHEMATICS II:

DATE:8/3/2023

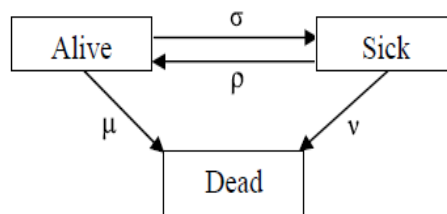
TIME:11:00-1:00 P.M

INSTRUCTIONS: Answer question one and any other two questions

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) Define the following terms.
- i. Profit signature. (2 marks)
 - ii. Off-period. (2 marks)
- b) Consider a person now aged exactly 25 whose annual salary rate is currently \$9,192. Estimate:
- i. His annual salary rate at exact age 65. (3 marks)
- If he dies at age 57 last birthday, determine the average values of
- ii. His annual salary rate at the moment of death. (3 marks)
- c) The profit signature of a 3-year assurance contract issued to a life aged 57 exact, with a premium payable at the start of each year of £500 is $(-250, 150, 200)$. Calculate the profit margin of the contract. (5 marks)
- Basis:
Mortality: AM92 Ultimate
Lapses: None
Risk discount rate :12% per annum
- d) Show that:
- $$H_x = D_{x+\frac{1}{2}} * Z_x \quad (5 \text{ marks})$$

- e) Suppose that the company's expenses in respect of this policy in the first month were 55% of annual premium plus £178, and that on average the mortality experience of all such policyholders was 58% of AM92 Ultimate.
- Death payments are made at the end of the month, after all charges have been deducted.
- Calculate the profit or loss to the company for that first month (ignoring the impact of interest on the non-unit fund). (5 marks)
- f) A three-state transition model is shown in the following diagram:



Assume that the transition probabilities are constant at all ages with $\mu = 2\%$, $v = 4\%$, $\rho = 1\%$ and $\sigma = 5\%$.

Calculate the present value of a sickness benefit of £2,000 p.a. paid continuously to a life now aged 40 exact and sick, during this period of sickness, discounted at 4% p.a. and payable to a maximum age of 60 exact. (5 marks)

QUESTION TWO (20 MARKS)

Under the rules of a pension scheme, a member may retire due to age at any age from exact age 60 to exact age 65.

On age retirement, the scheme provides a pension of 1/60th of Final Pensionable Salary for each year of scheme service, subject to a maximum of 40/60ths of Final Pensionable Salary. Only complete years of service are taken into account.

Final Pensionable Salary is defined as the average salary over the three-year period before the date of retirement.

The pension scheme also provides a lump sum benefit of four times Pensionable Salary on death before retirement. The benefit is payable immediately on death and Pensionable Salary is defined as the annual rate of salary at the date of death.

You are given the following data in respect of a member:

- Date of birth :1 January 1979
- Date of joining the scheme :1 January 2000
- Annual rate of salary at 1 January 2005: £50,000
- Date of last salary increase :1 April 2004

- a) Derive commutation functions to value the past service and future service pension liability on age retirement for this member as at 1 January 2005. State any assumptions that you make and define all the symbols that you use. (14 marks)
- b) Derive commutation functions to value the liability in respect of the lump sum payable on death before retirement for this member as at 1 January 2005. State any assumptions that you make and define all the symbols that you use. (6 marks)

QUESTION THREE (20 MARKS)

The decrement table extract below is based on the historical experience of a very large multinational company's workforce

<i>Age (x)</i>	<i>Number of employees</i> $(al)_x$	<i>Deaths</i> $(ad)_x^d$	<i>Withdrawals</i> $(ad)_x^w$
40	10,000	25	120
41	9,855	27	144
42	9,684		

Recent changes in working conditions have resulted in an estimate that the annual independent rate of withdrawal is now 75% of that previously used. Calculate a revised table assuming no changes to the independent death rates, stating your results to one decimal place.

QUESTION FOUR (20 MARKS)

- a) A friendly society provides the following benefits:
- On sickness, \$40 per week for the first 26 weeks and \$50 per week for the next 26 weeks. No sickness benefit is payable after age 65. (5 marks)
 - On attaining age 65 or immediately on earlier death, the sum of \$3,000.
- Members contribute \$1 per week, ceasing at age 65 or earlier death, but this is waived during periods of sickness (whether benefit is payable or not.) Calculate the reserve which should be held for a member aged 50, using the following basis:
- (5 marks)
- English Life Table No.12 - Males,
4% per annum interest,
Manchester Unity Sickness Experience 1893-97 (AHJ).
There is no waiting period, and off-periods are as in "Formulae and Tables for Actuarial Examinations".

- b) A life office is proposing to issue 3-year sickness benefit policies to lives aged 30. The benefits are \$50 per week during sickness within the next three years. There is no waiting period and the off-period is as in the Tables provided. Find the single premium on each of the following bases:
- mortality: English Life Table No.12 - Males
- interest:
- 4% p.a.,
 - 5% p.a.
- sickness: Manchester Unity 1893-97 (AHJ) expenses: none. (10 marks)

QUESTION FIVE (20 MARKS)

A 5-year unit-linked endowment assurance is issued to a male aged exactly 55. The expected year-end cashflows in the non-unit fund, $(NUCF)_t$ ($t = 1, \dots, 5$) per policy in force at the start of Year t are:

Year t	1	2	3	4	5
$(NUCF)_t$	-200	+20	+45	-60	+480

You are given:

Independent probabilities of mortality: AM92 Select

Independent probabilities of withdrawal: 0.1 for Years 1 and 2, 0.05 for Years 3 and 4, 0 for Year 5.

Withdrawals can occur at any time over the policy year.

- Calculate the net present value of profit at a risk discount rate of 10% *pa* assuming that the company holds no non-unit reserves. (10 marks)
The rate of interest earned on non-unit reserves is assumed to be 8% *pa*.
- Calculate the reserves that are required at times $t = 1, \dots, 4$ in order to zeroise future negative cashflows. (10 marks)