MACHAKOS UNIVERSITY

ISO 9001:2008 Certified

FIRST YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

SAC 100: PRINCIPLES OF ACTUARIAL SCIENCE
Supplementary Examinations
July 2019

NSTRUCTIONS TO CANDIDATES

Answer ALL the questions in Section A and ANY TWO Questions in Section B

Section A

QUESTION ONE (30 MARKS)

a) Define the following terms:
   i. Insurance (1 mark)
   ii. Underwriting (1 mark)
   iii. Risk (1 mark)

b) An investment is discounted for 28 days at a simple rate of discount of 4.5% per annum. Calculate the annual effective rate of interest. (3 marks)

c) Derive the formula for the present value of a unit n-year annuity due with payments being made once per year (2 marks)

b) A deposit of X is made into a fund which pays an annual effective interest rate of 6% for 10 years. At the same time an amount of X/2 is deposited into another fund which pays an annual effective rate of discount d for 10 years. The amounts of interest earned over the 10 years as equal for both funds. Calculate d. (4 marks)

c) The force of interest, $\delta(t)$, is a function of time and at any time t, measured in years, is given by the formula:

$$\delta(t) = 0.05 + 0.002t$$

Calculate the present value of an annuity that is paid continuously at a rate of $30e^{-0.01t+0.001t^2}$ units per annum from t=3 to t=10. [5 marks]

d)

i. Define the term risk. (1 mark)

ii. With aid of examples describe four types of risk according to the risk effects. (4 marks)

e) Calculate the annual effective rate of interest corresponding to a nominal rate of 11% per annum convertible half yearly (2 marks)

f) A student borrows Shs. 2,000 for 3 years at simple interest. The rate of interest is 8% per annum.

i. What is the accumulated amount at the end of year 3? (1 mark)

ii. Solve the problem in question (i) above using the compound interest method. (3 marks)

g) Calculate the present value of an annuity that pays $150 per annum annually in arrears forever using an annual effective rate of interest of 8%. (2 marks)

SECTION B

QUESTION TWO (20 MARKS)

a) Briefly describe the term 'Insurance' as a contract hence describe two examples of this type of life insurance contract. (3 marks)

b) Dollar bank has recently developed a risk management framework. During this process, they identified that they are prone to the risk of fire within their office premises as well as foreign exchange risk for their international transactions.

i. Briefly describe the risk management process they followed to develop the framework. (8 marks)

ii. Describe two risk management techniques that Dollar bank can employ to manage the two risks identified. (2 marks)

c) An insurance company offers a customer two payment options in respect of an invoice for £456. The first option involves 24 payments of £20 paid at the beginning of each month starting immediately. The second option involves 24 payments of £20.50 paid at the end of each month starting immediately. The customer is willing to accept a monthly payment schedule if the annual effective interest rate per annum he pays is less than 5%. Determine which, if any, of the payment options the customer will accept. (4 marks)

d) Calculate the rate of interest convertible monthly corresponding to a nominal rate of 11% per annum convertible three times a year (3 marks)

QUESTION THREE (20 MARKS)

a) Briefly describe four principles of insurance that govern the underwriting of insurance product. (4 marks)

b) ABC limited is a newly registered insurance company that will be underwriting insurance contracts to safeguard against morbidity risk. Briefly describe two types of insurance products they will be selling. (4 marks)

c) Describe briefly three factors that influence the level of interest rates. (3 marks)

d) Determine the annual effective rate of discount equivalent a rate of interest of 4% per annum convertible monthly. (3 marks)

e) A person wants to accumulate Shs.100,000 eight years from today to sponsor his son's education. If an investment plan offers him 8% compounded monthly, what amount must he invest today? (3 marks)

f) At a certain simple rate of interest KShs. 40,000 will accumulate to KShs. 52,500 after a certain period of time. Find the accumulated value of KShs. 31,000 at a simple rate of interest three fifths as great and over twice as long a period of time. (3 marks)

QUESTION FOUR (20 MARKS)

a) Calculate the present value of $100 over ten years at the following rates of interest/ discount:

i. A rate of interest of 12% per annum convertible monthly (2 marks)

ii. A rate of discount of 12% per annum convertible monthly (2 marks)

iii. A force of interest of 12% per annum (2 marks)

b) Assuming an interest rate of 12% p.a. convertible monthly:

i. Calculate the combined present value of an immediate annuity payable monthly in arrears such that payments are Shs.1,000 p.a. for the first 6 years and Shs 400 p.a. for the next 4 years, together with a lump sum of Shs 2,000 at the end of the 10 years (3 marks)

ii. Calculate the amount of the level annuity payable continuously for 10 years having the same present value as the payment in (i) above. (3 marks)

iii. Calculate the accumulated values of the first 7 years' payments at the end of the 7^{th} year for the payments in (i) and (ii) above. (3 marks)

c) Derive the formula for the present value of a continuously payable increasing annuity $\left(\overline{Ia}\right)_n$. (5 marks)

QUESTION FIVE (20 MARKS)

a) Calculate the present value of an annuity that pays $300 per annum monthly in arrears forever using an annual effective rate of interest of 6%. (4 marks)

b) The force of interest, $\delta(t)$, is a function of time and at any time t, measured in years, is given by the formula

$$\delta(t) = \begin{cases} 0.05 + 0.001t & 0 \le t \le 20 \\ 0.05 & t > 20 \end{cases}$$

i. Derive and simplify as far as possible expressions for v(t), where v(t) is the present value of a unit sum of money due at time t. (6 marks)

ii. (a) Calculate the present value of £100 due at the end of 25 years.
(b) Calculate the rate of discount per annum convertible quarterly implied by the transaction in part (ii)(a). (5 marks)

iii. A continuous payment stream is received at rate $30e^{-0,015t}$ units per annum between t = 20 and t = 25. Calculate the accumulated value of the payment stream at time t = 25. (5 marks)