MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF ARTS

SMA 430: NUMERICAL ANALYSIS II

DATE: 2/5/2019 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS:

Answer question **one** (**Compulsory**) and any other **two** questions.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) State the Weierstrass theorem (3 marks)

b) Find the least square polynomial approximation of degree two for the data in the table below. Compare with the exact and compute the total error. (7 marks)

i	x_i	y_i
1	1.0	1.84
2	1.1	1.96
3	1.3	2.21
4	1.5	2.45
5	1.9	2.94
6	2.1	3.18

c) Express the powers of x in terms of Chebyshev polynomials from x^0 to x^7. (6 marks)

d) Obtain the sixth Maclaurin's series for xe^x and use Chebyshev economization to obtain a lesser degree polynomial approximation while keeping the error less that 0.01 on $[-1,1]$. (6 marks)

e) Using Euler's method approximate the solution of $y' = te^{3t} - 2y, 0 \leq t \leq 1$, for $y(0) = 0$ with $h = 0.5$ (3 marks)

f) Using Newton's method with $x^{(0)} = (0,0,0)^t$, compute $x^{(2)}$ for the nonlinear system; (5 marks)

$$x_1{}^2 + x_2 - 37 = 0$$
$$x_1 - x_2{}^2 - 5 = 0$$
$$x_1 + x_2 + x_3 - 3 = 0$$

QUESTION TWO (20 MARKS)

a) Find the least squares approximating polynomial of lesser degree for $f(x) = x^2 - 2x + 3$ on the interval $[-1,1]$. (6 marks)

b) The Chebyshev polynomials are defined for $[-1,1]$ by $T_n(x) = \cos[n \cos^{-1} x]$;

i) Prove the recursion relation, $T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$ (4 marks)

ii) Prove the orthogonality property of Chebyshev polynomials. (5 marks)

$$\int_{-1}^{1} \frac{T_m(x)T_n(x)}{\sqrt{1-x^2}} dx = \begin{cases} 0\,; m \neq n \\ \dfrac{\pi}{2}\,; m = n \neq 0 \\ \pi\,; m = n = 0 \end{cases}$$

c) Determine the discrete least squares trigonometric polynomial approximation $S_2(x)$ on the interval $[-\pi, \pi]$ for $f(x) = \cos 2x$ (5 marks)

QUESTION THREE (20 MARKS)

a) Using the Modified Euler's method with $N = 4$

i) Approximate the solution of the initial value problem $y' = y - t^2 + 1$, $0 \leq t \leq 2, y(0) = 0.5$. (5 marks)

ii) Given that the exact solution is $y = 1 + t^2 + 2t - \frac{1}{2}e^t$ compare approximated results with the exact. (3 marks)

iii) Using the first four terms of the approximate solution, approximate $y(2.0)$, using the fourth-order Adams-Bashforth method. (2 marks)

b) Given that the initial value problem $y' = \frac{2}{t}y + t^2e^t, 1 \leq t \leq 2, y(1) = 0$ has an exact solution $y = t^2[e^t\ e]$

i) Using Runge-Kutta method of order four with $N = 4$ obtain an approximation to the solution of the problem. Compare with the exact. (8 marks)

ii) Using the first three terms of the approximation, approximate $y(1.75)$ using the Adams-Moulton three-step method. (2 marks)

QUESTION FOUR (20 MARKS)

Using Newton's method with $x^{(0)} = (0,0,0)^t$

Find the solution of the nonlinear system. Iterate until $\|x^k - x^{k-1}\|_\infty < 10^{-6}$.

$$3x_1 - \cos(x_2x_3) - \frac{1}{2} = 0$$
$$4{x_1}^2 - 625{x_2}^2 + 2x_2 - 1 = 0$$
$$e^{-x_1x_2} + 20x_3 + \frac{10\pi - 3}{3} = 0$$

QUESTION FIVE (20 MARKS)

Apply the linear shooting technique with $N = 4$ to the boundary value problem

$$y'' = \frac{-2}{x}y + \frac{2}{x^2}y + \frac{\sin(\ln x)}{x^2}$$

For $1 \le x \le 2$ with $y(1) = 1$ and $y(2) = 2$. Compare your results with the exact solution

$$y = 1.1392070132x - \frac{0.03920701320}{x^2} - \frac{3}{10}\sin(\ln x) - \frac{1}{10}\cos(\ln x)$$