



MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL
SCIENCE

FOURTH YEAR SPECIAL/ SUPPLEMENTARY EXAMINATION FOR
BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF ARTS

SMA433-PARTIAL DIFFERENTIAL EQUATIONS II

DATE:

TIME:

INSTRUCTIONS:

Answer **Question One** and **Any Two** Other Questions.

QUESTION ONE (COMPULSORY) (30MARKS)

- a) Evaluate the solution to the following initial value problem

$$u_{xx} = 4xy + e^x$$

with initial conditions

$$u(0, y) = y, u_x(0, y) = 1 \quad (5 \text{ marks})$$

- b) Solve the equation $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^4} = 2 \frac{\partial^4 z}{\partial x^2 \partial y^2}$ (5 marks)

- c) Find the particular integral of the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial z}{\partial y} = 4e^{2x+4y} \quad (6 \text{ marks})$$

- d) Consider the equation

$$u_{xx} + 4u_{xy} + 4u_{yy} = 0. \text{ Classify the equation and find the general solution. 6 marks}$$

- e) A circular cylinder of radius a has its surface kept at a constant temperature θ_0 . if the internal temperature is zero throughout the cylinder, prove that for $t > 0$,

$$\theta(r, t) = \theta_0 \left\{ 1 - \frac{2}{\omega} \sum_{n=1}^{\infty} \frac{J_0(\xi_n r)}{\xi_n J_n(\xi_n a)} e^{-\xi_n^2 kt} \right\}$$

Where $\pm \xi_0, \pm \xi_1, \dots, \pm \xi_n$ are roots of $J_0(\xi_n) = 0$ (8 marks)

QUESTION TWO (20 MARKS)

- a) Obtain a partial differential equation of the first order by eliminating the arbitrary function

$$u = e^{-y} f(x) + e^x g(y) \quad (4 \text{ marks})$$
- b) Reduce the equation $r - x^2 t = 0$ to canonical form (8 marks)
- c) Show that Laplace's equation in three dimensions

$$u_{xx} + u_{yy} + u_{zz} = 0 \quad (8 \text{ marks})$$

QUESTION THREE (20 MARKS)

- a) i) Find the characteristics of equation $r - yt = 0$
 ii) Show under which condition the curve will be a characteristic of the partial differential equation $p^2 r + 3 p q s + 2 q^2 t = 0$ (5 marks)
- b) Find the complete solution of the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = (x - y) \quad (5 \text{ marks})$$
- c) Reduce the equation $r + 2s + t = 0$ into canonical form and solve completely the corresponding equation. (10 marks)

QUESTION FOUR (20 MARKS)

- a) If i.) $(\alpha_r D_1 + \beta_r D_2 + \rho_r)$, is a linear factor of the linear differential operator $F(D_1 D_2)$
 i.) $\phi_r(\xi)$, is arbitrary function of one variable ξ
 ii.) $\alpha_r \neq 0$
 Prove that $z_r = \sum_{r=1}^n \exp\left(-\frac{\rho_r}{\alpha_r} x\right) \phi_r(\beta_r x - \alpha_r y)$ is the solution of the reduced equation $F(D_1 D_2)z = 0$ (10 marks)
- b) Find the general solution of the partial differential equation

$$x^2 z_{xx} - 2xyz_{xy} - y^2 z_{yy} - xz_x + yz_y - 5z = 7x - 5y \quad (10 \text{ marks})$$

QUESTION FIVE (20 MARKS)

Given the partial differential equation

$$r - t = 0 \dots\dots (i)$$

$$\text{and five real functions } \left. \begin{array}{l} x = t \\ y = 0 \\ z = t^2 + t^4 \end{array} \right\} \dots\dots(ii) \text{ and}$$
$$\left. \begin{array}{l} p = 2t + 4t^3 \\ q = 4t \end{array} \right\} \dots\dots(iii).$$

Show that the five real functions form a strip hence interpret geometrically initial value problem in which we seek a solution of the form $z = z(x, y)$