



MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (ECONOMICS AND STATISTICS)

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

SMA 363 TESTS OF HYPOTHESIS I

DATE:

TIME:

INSTRUCTIONS

Answer Question ONE and ANY OTHER TWO

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) Explain briefly what is meant by hypothesis testing as used in statistics. (2 marks)
- b) Outline the general procedure used for hypothesis testing. (4 marks)
- c) Differentiate between Type I and Type II errors as used in hypothesis testing. (4 Marks)
- d) Use the Neyman-Pearson Lemma to obtain the best critical region for testing $\theta = \theta_0$ against $\theta = \theta_0 > \theta_1$, in the case of a normal population $N(\theta, \sigma^2)$, where σ^2 is known. (7 marks)
- e) A new gene has been identified that makes carriers particularly susceptible to a particular degenerative disease. In a random sample of 250 adult males born in Kenya, 8 were found to be carriers of the disease. Test whether the proportion of adult males born in Kenya carrying the gene is less than 10%. (5 marks)
- f) Until recently, most Chief Executive Officers (CEOs) of companies in a certain country have been men. However, as government encourages companies to increase the number of female CEOs, a number of companies now have female CEOs. A researcher decided to examine the success of female CEOs of medium-size companies by comparing their base salaries (excluding bonuses and outliers) with those of their male counterparts. She took a random sample of 100 female and 100 male CEOs and recorded their salaries for the preceding year. The average salary for the female CEOs was \$402 500 and for the male CEOs was \$430000. From past records, we have information that CEO incomes are normally distributed with a standard deviation of

\$7000 for both females and males. At the 5% level of significance, is there enough evidence to support the claim that average salaries of female CEOs are lower than average salaries of male CEOs, of medium-size companies? (8 marks)

QUESTION TWO (20 MARKS)

- a) A manufacturer of television screens has a production line that used to produce an average of 200 screens per day. Because of recently introduced government regulations, a new safety device is installed, which the manufacturer believes will reduce the average daily output. After installation of the safety device, a random sample of 15 days' production was recorded, as follows:

186	206	190	202	182
210	192	188	202	176
196	188	202	184	190

Assuming that the daily output is normally distributed, is there sufficient evidence to allow the manufacturer to conclude that the average daily output has decreased following installation of the safety device? $\alpha = 0.05$. (5 marks)

- b) Let $X_1, \dots, X_n$ be a random sample from a Poisson distribution with mean λ . Derive the most powerful test for testing $H_0: \lambda = 3$ against $H_a: \lambda = 6$. (7 marks)
- c) If $X \geq 1$ is the critical region for testing $H_0: \theta = 2$ versus $H_1: \theta = 1$ on the basis of a single observation from the population

$$f(x, \theta) = \theta \exp(-\theta x), 0 \leq x \leq \infty$$

Determine the type I and type II errors. (8 marks)

QUESTION THREE (20 MARKS)

- a) The average blood pressure for a control group C of 10 patients was 77.0mmHg. The average blood pressure in a similar group T of 10 patients on a special diet was 75.0mmHg. Carry out a statistical test to assess whether the variance in the two populations can be considered to be equal. Take $\sum_{i=1}^{10} C_i^2 = 59,420$, $\sum_{i=1}^{10} T_i^2 = 56,390$ and $\alpha = 0.05$. (6 marks)
- b) Because of the impact of the global economy on a high-wage country such as the United States, it is claimed that the domestic content in manufacturing industries fell between 1977 and 1997. A survey of 36 randomly picked U.S. companies gave the proportion of domestic content total manufacturing in 1977 as 0.37 and in 1997 as 0.36. At the 1% level of significance, test the claim that the domestic content really fell during the period 1977–1997. (6 marks)
- c) Let $X_1, \dots, X_n$ be a random sample from a $N(\mu, \sigma^2)$. Assume that σ^2 is known. Determine an appropriate likelihood ratio test to test $H_0: \mu = \mu_0$ vs $H_a: \mu \neq \mu_0$ at level α . (8 marks)

QUESTION FOUR (20 MARKS)

- a) Differentiate between a Most powerful test and the Uniformly most powerful test. (4 marks)

- b) The intelligence quotients (IQs) of 17 students from one area of a city showed a sample mean of 106 with a sample standard deviation of 10 whereas the IQs of 14 students from another area chosen independently showed a sample mean of 109 with a sample standard deviation of 7. Is there a significant difference between the IQs of the two groups at $\alpha = 0.02$? Assume that the population variances are equal. (8 marks)
- c) A random variable X is believed to follow an $Exp(\lambda)$ distribution. In order to test the null hypothesis $\mu = 20$ against the alternative $\mu = 30$, where $\mu = \frac{1}{\lambda}$, a single value is observed from the distribution. If this value is less than 28, H_0 is accepted, otherwise H_0 is rejected. Find the probabilities of
- a Type I error
 - a Type II error
- (8 marks)

QUESTION FIVE (20 MARKS)

- a) A physician claims that the variance in cholesterol levels of adult men in a certain laboratory is at least 100. A random sample of 25 adult males from this laboratory produced a sample standard deviation of cholesterol levels as 12. Test the physicians claim at 5% level of significance. (4 marks)
- b) Ten persons were appointed in an officer cadre in an office. Their performance was noted by giving a test and the marks were recorded out of 100. They were given 4 months training and a test was held and marks were recorded out of 100.

Before	80	76	92	60	70	56	74	56	70	56
After	84	70	96	80	70	52	84	72	72	50

Do the data provide sufficient evidence to support the claim that the employees benefited from the training? Take $\alpha = 0.05$. (8 marks)

- c) The table shows the midterm and final examination test scores for 10 students from a differential equations class, where x denotes the midterm scores and y denotes the final scores.

x	68	87	75	91	82	77	86	82	75	79
y	74	89	80	93	88	79	97	95	89	92

At 95% confidence level, test whether X and Y are independent. (8 marks)