MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FIRST YEAR SECOND SEMESTER EXAMINATIONS FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF SCIENCE (MATHEMATICS)

SST 102: DISCRETE MATHEMATICS

DATE: 07/05/2019 **TIME: 11:00 – 1:00 PM**

INSTRUCTIONS

Answer Question One and any other two questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Explain the meaning of the following terms

(i) transitive binary operation on a set A, (2 marks)

(ii) relatively prime numbers $a, b \in \mathbb{Z}$. (2 marks)

b) Define a relation $\sim$ on the set $\mathbb{Z}$ as follows: $a \sim b$ if and only if $a - b$ is divisible by 6.

(i) Determine if $\sim$ is an equivalence relation. (4 marks)

(ii) Find the equivalence class of 3. (3 marks)

c) Determine if the following function from $\mathbb{Z}$ to $\mathbb{Z}$ is injective or surjective

$$f(n) = n^2 + 15.$$

Justify your answer. (5 marks)

d) Let n and d be positive integers. How many integers not exceeding n are divisible by d? (4 marks)

e) Let A and B be sets with m and n elements respectively. Prove that there are n^m functions from A to B respectively. (5 marks)

f) Solve the following reccurence relation

$$a_n = 6a_{n-1} - 9a_{n-2}$$

(5 marks)

QUESTION TWO (20 MARKS)

a) Solve the following linear congruence equation

$$9x \equiv 15 (\mathrm{mod}\ 12)$$

(5 marks)

b) Let a, b and c be integers. Show that

(i) If $a|b$ and $a|c$ then $a|(b+c)$ (3 marks)

(ii) If $a|b$ and $a|bc$ for all integers c (2 marks)

(iii) If $a|b$ and $b|c$, the $a|c$. (2 marks)

c) Let a, b and c be positive integers such that $\gcd(a, b) = 1$ and $a|bc$. Prove that $a|c$. (4 marks)

d) In a particular general election, there are eight candidates vying for a given position. What is the minimum number of valid votes cast so as to ensure that at least one candidate receives not less than 3,345,609 votes? Justify your answer. (4 marks)

QUESTION THREE (20 MARKS)

a) Use the Euclidean algorithm

(i) to find $\gcd(662,414)$. (3 marks)

(ii) Using (i) above or otherwise express $\gcd(662,414)$ in the form

$$d = 662s + 414t$$

where s and t are integers. (3 marks)

b) Prove that if n is an integer not divisible by 3 then $n^2 \equiv 1 (\mathrm{mod}\ 3)$. (5 marks)

c) Consider all integers from 1 up to and including 300. Find the number of them that are divisible by:

(i) at least one of 3, 5; (2 marks)

(ii) by 5, but by not by 3 ; (2 marks)

d) Suppose that there are 3 distinct courses in computational mathematics, 5 distinct courses in statistics and 8 distinct courses in fluid mechanics. How many distinct ways can a student choose 2 courses in computational mathematics, 3 courses in statistics and 4 courses in fluid mechanics? (5 marks)

QUESTION FOUR (20 MARKS)

a) Suppose that $A = \mathbb{R} \setminus \{-2\}$ and $B = \mathbb{R} \setminus \{1\}$. Consider the function $f: A \to B$ defined by

$$f(x) = \frac{x-1}{x+2}.$$

(i) Prove that f is an injective function. (4 marks)

(ii) Determine whether f is a surjective function. (4 marks)

b) The chairs of a lecture hall are to be labeled with an uppercase letter followed by a positive integer more than 10 but less than 100. What is the maximum number of chairs that can be labeled this way? (2 marks)

c) Each user on a computer system has a password which is six to eight characters long. Each character in a generated password can be a lowercase alphabetic letter or a numeric digit. Each password must contain at least one digit. Determine the possible number of unique passwords that can be generated. (5 marks)

d) How many solutions does the equation $x_1 + x_2 + x_3 = 13$ have where x_1, x_2 and x_3 are non-negative integers. (5 marks)

QUESTION FIVE (20 MARKS)

a) Let $\mathbb{Z}$ denote the set of integers. Consider the functions $f: \mathbb{Z} \to \mathbb{Z}$ and $g: \mathbb{Z} \to \mathbb{Z}$ given respectively by $f(n) = 2n + 3$ and $g(n) = 3n + 2$. What is

(i) the composition $g \circ f$, (2 marks)

(ii) the composition $f \circ g$. (2 marks)

b) Consider the sequence $\{a_n\}$ defined by the following recurrence relation

$$a_n = 3a_{n-1} - 2a_{n-2}.$$

Solve this recurrence relation subject to the initial condition $a_0 = 1, a_1 = 3$. (6 marks)

c) Prove the Pascal's identity - i.e.

$$\binom{n+r}{r} = \binom{n}{r-1} + \binom{n}{r}$$

(7 marks)

d) Let S be a set with n elements. Prove that the total number of subsets of S is given by 2^n. (3 marks)