



MACHAKOS UNIVERSITY COLLEGE

(A Constituent College of Kenyatta University)
University Examinations for 2015/2016

SCHOOL OF BUSINESS AND ECONOMICS

DEPARTMENT OF ECONOMICS

SECOND SEMESTER EXAMINATION FOR DEGREE IN
BACHELOR OF ECONOMICS AND STATISTICS
BACHELOR OF ECONOMICS AND FINANCE
BACHELOR OF ARTS (MATHEMATICS)
BACHELOR OF ARTS (ECONOMICS)
BACHELOR OF ECONOMICS

EES 200: MATHEMATICS FOR ECONOMISTS II

Date: 5/8/2016

Time: 8:30 – 10:30 AM

INSTRUCTIONS

Attempt question **ONE** and any other **THREE** questions.

QUESTION ONE (Compulsory)

- a) Differentiate between comparative statics analysis and optimization problems (3 marks)
- b) Given the following demand and supply functions for a one commodity market.

$$Q_d = a - bP$$

$$Q_s = -c + dP$$

$$Q_d = Q_s$$

Do a complete comparative static analysis of the model and graphically represent your results (7 marks)

- c) Consider the following production function $Q(k, l) = 3k^3 - 5kl^2 + l^3$
- Test whether the function is homogenous and comment on its returns to scale (3 marks)
 - Proof Euler's theorem for the production function in c(i) above (3 marks)
 - Proof the young's theorem for the production function in c(i) above (3 marks)
- d) A three product model is given as
- $$Q_{d1} = 50 - 2P_1 + 5P_2 - 3P_3 \quad Q_{s1} = 8P_1 - 5$$
- $$Q_{d2} = 22 + 7P_1 - 2P_2 + 5P_3 \quad Q_{s2} = 12P_2 - 5$$
- $$Q_{d3} = 17 + P_1 + 5P_2 - 3P_3 \quad Q_{s3} = 4P_3 - 1$$
- Determine the equilibrium prices and quantities for the three products using Matrix algebra (7 marks)
- e) Evaluate $\int (2x^2 + 3)^5 dx$ (4 marks)

QUESTION TWO

- a) Find $\frac{\partial y}{\partial x_i}$ for $i = 1, 2$ in the following functions
- $y = f(x_1, x_2) = 10(x_1^2 + x_2)^3 x_1 x_1^{0.5}$
 - $y = f(x_1, x_2) = -5e^{\left(\frac{2x_1^3}{5x_2}\right)}$
 - $y = f(x_1, x_2) = 15 \ln(5x_1^3 + x_2^5)$
 - $y = f(x_1, x_2) = 15 \log(5x_1^3 + x_2^5)$ (8 marks)
- b) Find the total differential of the following utility functions
- $U = x_1^2 + x_2^3 + x_1 x_2$ (2 marks)
 - $U = x_1^a x_2^b$ (3 marks)
- c) Find the rank of $\begin{bmatrix} 7 & 6 & 3 & 3 \\ 0 & 1 & 2 & 1 \\ 8 & 0 & 0 & 8 \end{bmatrix}$ (7 marks)

QUESTION THREE

- a) Consider the following national income model for an open economy with government

$$Y = C + I_0 + G_0 + X_0 - M$$

$$C = c_0 + c_1(Y^d) \text{ where } c_0 > 0 \quad 0 < c_1 < 1$$

$$T = t_0 + t_1 Y \quad t_0 > 0 \quad 0 < t_1 < 1$$

$$Y^d = Y - T$$

$$M = m_0 + m_1 Y$$

$$I = I_0 \quad \text{and} \quad G = G_0$$

. Find

- i) The expression for the equilibrium value of $\bar{Y}$ (5 marks)
 - ii) The total derivative of $\bar{Y}$ (3 marks)
 - iii) Autonomous investment multiplier $\left(\frac{d\bar{Y}}{dI_0}\right)$ and the tax rate multiplier $\left(\frac{d\bar{Y}}{dt_1}\right)$ (4 marks)
- b) The marginal propensity to consume (MPC) for some economy is 0.75. determine the consumption function for the economy given that $C = 100$ and $Y = 0$ (5 marks)
- c) A production function is given as $Q = 80K^{0.2}L^{0.5}$. Determine the marginal product of labor, the marginal product of capital and test whether the marginal products are increasing or decreasing (4 marks)

QUESTION FOUR

- a) A firm has the following total cost function and the demand function

$$C = \frac{1}{3}Q^3 - 7Q^2 + 111Q + 50$$

$$Q = 100 - p$$

- i) Write the expression of the total revenue (3 marks)
- ii) Find the profit maximizing level output Q^* (4 marks)

- iii) Find the maximum level of profits and Proof that the level output Q^* obtained in (ii) above meets the condition of profit maximization (3 marks)
- b) Consider the following input – output table for an economy with three sectors

	Agriculture	Energy	Manufacturing	Final demand	Gross production
Agriculture	2	4	0	4	10
Energy	0	4	8	8	20
Manufacturing	3	3	9	15	30
Primary inputs					
Total value of inputs	10	20	30		

- i) Complete the table (1 marks)
- ii) Determine GNP by factor payment approach (2 marks)
- iii) Determine the
- a) coefficient matrix (2 marks)
- b) Leontief Matrix (2 mark)
- c) Leontief Inverse (3 marks)

QUESTION FIVE

- a) Evaluate

i) $\int \frac{x}{x^2+1} dx$ (4 marks)

ii) $\int e^{2x^2} dx$ (4 marks)

- b) Supply and Demand functions for a particular market are $Q = -16 + 3p^2$ and $Q = 35 - 6P^2$. Calculate the producer and consumer surplus for the market (6 marks)
- c) For the following two demand functions, compute the four partial elasticities of the demand and state whether the two commodities are substitutes or complements given that

$$Q_1, Q_2, P_1, P_2 > 0$$

$$Q_1 = 7 - 2P_1 - P_2$$

$$Q_2 = 23 - P_1 - 3p_2$$
 (6 marks)