MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

SECOND YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (TELECOMMUNICATION INFORMATION TECHNOLOGY)

SPH 205: MATHEMATICAL PHYSICS

DATE: 17/8/2021 **TIME: 2.00-4.00 PM**

INSTRUCTIONS:

- The paper consists of **two** sections.
- Section **A** is **compulsory** (30 marks).
- Answer any **two** questions from section **B** (each 20 marks).

QUESTION ONE (30 MARKS)

a) The instantaneous voltage v in a capacitive circuit is related to time t by the equation $v = Ve^{-t/C.R}$ where *V*, *C* and *R* are constants. Determine *v*, when $t = 30\text{x}10^{-3}$ sec, $\text{C} = 10\text{x}10^{-6}\,F$, $R = 47\text{x}10^{3}\,\Omega$, and $V = 200V$. (2 marks)

b) Determine the value of $3e^{-1}$, using the power series for e^{x}. (3 marks)

c) Plot a graph of $y = \frac{1}{3}e^{-2x}$ over the range $x = -1.5$ to $x = 1.5$. Determine from the graph the value of *y* when $x = -1.2$ and the value of *x* when $y = 1.4$. (4 marks)

d) Given $20 = 60(1 - e^{-t/2})$, determine the value of *t*, correct to 3 significant figures. (3 marks)

e) Express $(5, 124^{0})$ in Cartesian coordinates. (3 marks)

f) Find the volume of the solid formed when the plane figure bounded by $r = 2a\cos\theta$ and the radius vectors at $\theta = 0$ and $\theta = \pi/2$, rotates about the initial line. (3 marks)

g) A drilling machine is to have 6 speeds ranging from 50 rev/min to 750 rev/min. if the speeds form a geometric progression, determine their values, each correct to the nearest whole number. (4 marks)

h) The radius of a cylinder is reduced by 4% and its height is increased by 2%. Determine the approximate percentage change in:

i. Its volume and, (3 marks)

ii. Its curved surface area, neglecting the products of small quantities. (2 marks)

i) i. Find the gradient of the scalar field; $f(x, y, z) = xy^2 - yz$ (2 marks)

ii. Find the divergence of the vector field $F = 3x^2\hat{i} - 6xy\hat{j}$. Explain the result (2 marks)

iii. Find the curl of the vector field $F = y^3\hat{i} + xy\hat{j} - z\hat{k}$. (2 marks)

QUESTION TWO (20 MARKS)

a) The decay of voltage, v volts, across a capacitor at time t seconds is given by $v = 250e^{-t/3}$.

i. Draw a graph showing the natural decay curve over the first 6 seconds. (5 marks)

ii. From the graph, find: i) the voltage after 3.4 s, and ii) the time when the voltage is 150 V. (3 marks)

b) Expand $\frac{1}{(4-x)^2}$ in ascending powers of x as far as the term in x3, using the binomial theorem.

(4 marks)

c) Find the limits of x for which the expansion in (b) is true. (1 mark)

d) Find the angle between the vectors; $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$. (4 marks)

e) Find the vector area of the surface of the hemisphere $x^2 + y^2 + z^2 = a^2, z \geq 0$, by evaluating the line integral $S = \frac{1}{2}\oint_C r \text{ x } dr$ around its perimeter. (3 marks)

QUESTION THREE (20 MARKS)

a) The current i in amperes flowing in a capacitor at time t in seconds is given by $i = 8.0\left(1 - e^{-t/C.R}\right)$, where $R = 25x10^3\,\Omega$, and $C = 16x10^{-6}\,F$. Determine;

i. The current i after 0.5 seconds (3 marks)

ii. The time, to the nearest millisecond, for the current to reach 6.0A. (3 marks)

iii. Sketch the graph of the current against time, over the range $0 \le t \le 2$. (4 marks)

b) Show that $\nabla.(\nabla\phi \times \nabla\varphi) = 0$, where ϕ and φ are scalar fields. (3 marks)

c) Given the scalar field $\phi = xy^2z^3$. Find:

i. The gradient (2 marks)

ii. The Laplacian of the scalar field (2 marks)

d) A particle of mass m with position vector r relative to some origin O experiences a force F, which produces a torque (moment) $\tau = \vec{r}\ \text{x}\ \vec{F}$ about O. The angular momentum of the particle about O is given by $\vec{L} = \vec{r}\ \text{x}\ \text{m}\vec{V}$, where V is the particle's velocity. Show that the rate of change of angular momentum is equal to the applied torque. (3 marks)

QUESTION FOUR (20 MARKS)

a) The temperature θ_2 of a winding which is being heated electrically at time t is given by; $\theta_2 = \theta_1\left(1 - e^{-t/\tau}\right)$, where θ_1 is the temperature (in degree Celsius) at time $t = 0$, and τ is a constant. Calculate:

i. θ_1, correct to the nearest degree, when $\theta_2 = 50\ {}^0C$, $t = 30$ s, and $\tau = 60$ s. (2 marks)

ii. The time t, correct to 1 decimal place, for θ_2 to be half the value of θ_1. (3 marks)

b) Find the surface area generated when the arc of the curve $r = ae^{\theta}$ between $\theta = 0$ and $\theta = \pi/2$, rotates completely about the initial line. (6 marks)

c) Find the volume V of the parallelepiped with sides

$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = 4\hat{i} + 5\hat{j} + 6\hat{k}$ and $\vec{c} = 7\hat{i} + 8\hat{j} + 10\hat{k}$. (5 marks)

d) Expand $f(x) = \cos x$ as a Taylor series about $x = \pi/3$. (4 marks)

QUESTION FIVE (20 MARKS)

a) The first, twelfth, and last term of an arithmetic progression are; 4, 31.5, and 376.5 respectively. Determine;

i. The number of terms in the series, (4 marks)

ii. The sum of all the terms, and (2 marks)

iii. The 80th term. (1 mark)

b) The resonant frequency of a vibrating shaft is given by: $f = \frac{1}{2\pi}\sqrt{\frac{k}{I}}$, where k is the stiffness and I is the inertia of the shaft. Use the binomial theorem to determine the approximate percentage error in determining the frequency using the measured values of k and l when the measured value of k is 4% too large and the measured value of I is 2% too small. (5 marks)

c) Show that the area of a region R enclosed by a simple closed curve C is given by $A = \frac{1}{2}\oint_C (xdy - ydx) = \oint_C xdy = -\oint_C ydx.$ Hence calculate the area of the ellipse $x = a\cos\phi,\ y = b\sin\phi.$ (5 marks)

d) Find the angle between the vectors; $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$. (3 marks)