MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FIRST YEAR FIRST SEMESTER EXAMINATION FOR

MASTER OF SCIENCE (MATHEMATICAL MODELING AND COMPUTATION)

SMA 802: MATHEMATICAL METHODS FOR DIFFERENTIAL EQUATIONS

DATE: 15/5/2019 **TIME: 2:00 – 5:00 PM**

INSTRUCTIONS TO CANDIDATES

Answer question ONE and any other TWO questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Find the Laplace transform of the following functions

i) $F(t) = e^{at}$ (4 marks)

ii) $F(t) = \cos bt$, for $t > 0$. (4 marks)

b) Using the series definition for $J_p(x)$, show that

$$\frac{d}{dx}\left(x^{-p}J_p(x)\right) = -x^p J_{p+1}(x)$$ (6 marks)

c) Solve the differential equation $y'' - 2x^2 y' - y = 0$ in powers of x. (8 marks)

d) Solve the partial differential equation

$$\frac{\delta^2 u}{\delta x^2} - \frac{\delta^2 u}{\delta x \delta y} - \frac{\delta^2 u}{\delta y^2} = x + y$$ (8 marks)

QUESTION TWO (20 MARKS)

a) Prove the following recurrence relations for the Bessel functions

i. $\frac{d}{dx}\left(J_0\left(x\right)\right) = -J_1\left(x\right).$

ii. $\frac{d}{dx}\left(x^p J_p\left(x\right)\right) = x^p J_{p-1}\left(x\right)$ (8 marks)

b) Show that the alternative solution of the Bessel's differential equation in terms of $J_p\left(x\right)$ is

$$y = AJ_n\left(x\right) + BJ_n\left(x\right)\int \frac{1}{xJ_n^2\left(x\right)}dx$$

Where A and B are arbitrary constants (12 marks)

QUESTION THREE (20 MARKS)

a) Determine inverse Laplace transform of the function

$$f\left(s\right) = \frac{1}{s\left(s^2+1\right)}$$ (6 marks)

b) Solve the partial differential equation

$$\frac{\delta^2 y}{\delta x^2} = \frac{\delta^2 y}{\delta t^2}.$$

$$y\left(0,t\right) = 0, 0 \le t \le \infty$$

$$y\left(\pi,t\right) = 0, 0 \le t \le \infty$$

$$y\left(x,t\right) = \sin 3x, 0 \le x \le \pi$$

$$\frac{\delta y\left(x,0\right)}{\delta t} = 0, 0 \le x \le \pi$$

By the method of separation of variables. (14 marks)

QUESTION FOUR (20 MARKS)

a) Solve the linear equation

$$y' + y = 2\sin t$$

$$y\left(0\right) = -1$$

By using Laplace transform. (8 marks)

b) Let F be a real valued function which is continuous for $t \geq 0$ and of exponential order $e^{\alpha t}$. If F' is the derivative of F, and is piecewise continuous in every finite closed interval $a \leq t \leq b$, then prove the Laplace transform of $F'(t)$ is given by

$\ell\{F'(t)\} = s\ell\{F(t)\} - F(0)$ for $s > \alpha$. (12 marks)

QUESTION FIVE (20 MARKS)

Find the series solution of the differential equation $x^2 y'' - xy' - \left(x^2 + \frac{5}{4}\right)y = 0$