

MACHAKOS UNIVERSITY

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR, FIRST SEMESTER EXAMINATIONS

For Diploma in building and Civil Engineering. (TVET)

Date

Time **3 hours**

Instructions

The paper consists of **EIGHT** questions. Answer any **FIVE** questions.

ALL questions carry equal marks.

Show all your working

1. (a) Simplify the expressions;

i)
$$\frac{(1-x)^{\frac{1}{2}} - x(1-x)^{-\frac{1}{2}}}{1-x}$$

ii)
$$\frac{\log 729 - 4\log 3 + 2\log 27}{\log 243 - \log 27 + \log 9}$$
 without using logarithm tables (7marks)

- b) Solve the equations;

i) $\log_2 x + 2\log_4(x+1) = 1$

ii) $4^x = 2 + 16^{\frac{x}{4}}$ (13 marks)

2. a) Determine the values of p, q, and r such that $4x^2 - 3x + 12 = p(x + q)^2 + r$ (5 marks)

- b) The roots of the equation $ax^2 + bx + c = 0$ are α and $\alpha + 2$. Prove that

$b^2 = 4(a^2 + ac)$. (7marks)

- c) Solve the following simultaneous equations

$x + 2y - z = 1$

$x + 3y - 2z = 0$

$x + y + z = 4$

Use the method of substitution to solve the equation. (8 marks)

3) a) Simplify the expression $5 \times 4^{3n+1} - 20 \times 8^{2n}$ (4 marks)

b) Find the values of:

i) $\frac{\log 15625}{\log 25} - 2$

ii) $\frac{8^{\frac{2}{3}} + 4^{\frac{3}{2}}}{16^{\frac{3}{4}}}$ (6 marks)

c) Given that $2\log 8N = p$, $\log 22N = q$ and that $q - p = 4$, determine the value of N.

(10 marks)

4. a) Given that $\sin A = \frac{12}{13}$ and $\cos B = \frac{4}{5}$ where A is obtuse and B is acute, determine the values of ;

i) $\sin (A - B)$

ii) $\tan (A + B)$

b) Prove the identities:

i) $\frac{1 - \cos \theta}{\sin \theta} + \frac{\sin \theta}{1 - \cos \theta} = 2 \operatorname{cosec} \theta$

ii) $\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$ (8 marks)

c) Given $t = \tan 22 \frac{1}{2}^\circ$

i) Show that $\tan 45^\circ = \frac{2t}{1 - t^2}$;

ii) Hence solve the equation:

$t^2 + 2t - 1 = 0$, leaving your answer in surd form. (7 marks)

5.a) Express in polar co-ordinates the position :

i) $P_1(3, 4)$ ii) $P_2(-5, -8)$ (6 marks)

b) obtain the Cartesian equations of;

i) $r = 5(1 + 2 \cos \theta)$

ii) $r = a \tan \theta$ (7 marks)

c) Find the cartesian equations of the loci;

i) $x = t^2 + 4$ and $y = t - 3$

ii) $x = 5\cos\theta$ and $y = 4\sin\theta$ (7 marks)

6.a) The second term of an arithmetical progression is 15 and the fifth term is 21.

Determine the :

- i) Common difference
- ii) First term
- iii) The sum of the first ten terms (7 marks)

b) Find the difference between the sums of the first ten terms of the arithmetical progressionS

whose first terms are 12 and 8, and whose common differences are 2 and3 respectively. (5 marks)

c) Given the first, third and sixth terms of an arithmetical progression are in geometrical progression, find the common ratio of the geometric progression. (8 marks)

7.a) i) Find the sum of the integers between 1 and 100

ii)Determine the sum of the first six terms of the geometrical progression

$5 + 15 + 45 + \dots$ (12 marks)

b) A sum of Ksh 4000 is deposited in a bank account at a simple interest rate of 3%

per annum. Determine, using AP's and GP's, the:

- i) Amount after ten years;
 - ii) Number of years required for the amount in (i) to be realized at compound
- Interest rate of 2% per annum (8 marks)

8. a) Solve the simultaneous equations $2x + y = 7$ and $3x - 2y = 5$ graphically (5 marks)

b) Obtain the solution to the following equations graphically

$y = 2e^x$ and $y = 3x + 4$ for $-1 \leq x \leq 4$ (8 marks)

c) Draw the graph of $y = 2x^2 - 4x - 16$ for $-3 \leq x \leq 5$. Hence use your graph to solve the equation $2x^2 - 6x - 10 = 0$. (7 marks)