



MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS)

SMA 333: FLUID MECHANICS I

DATE: 22/1/2021

TIME: 2.00-4.00 PM

INSTRUCTIONS TO CANDIDATES

Attempt question *one (compulsory)* and any other *two questions*.

QUESTION ONE (30 MARKS)

- a) A plate at a distance of 0.2cm from a fixed point. The plate moves 1 m/sec and requires a force of 40 dyres/cm² to maintain that speed. Determine the coefficient of viscosity of the fluid between the plates. (5 marks)
- b) A liquid compressed in a cylinder has a volume of 0.4cc at 6.8×10^7 dynes/cm² and a volume of 1.396×10^8 dynes/cm². Determine its bulk modulus. (5 marks)
- c) Prove that $C_p - C_v = \rho$ (5 marks)
- d) Calculate the velocity field of a spherically symmetric flow whose stream function is
$$\psi(r, \theta) = \frac{Ua^3}{2r} \sin^3 \theta - \frac{Ur^2}{2} \sin^2 \theta$$
 (5 marks)
- e) The velocity potential of a 2-d motion is $\phi = xyk..$ Determine the stream lines. (5 marks)

- f) Determine whether the fluid with vorticity component $u = \frac{-2xyz}{(x^2 + y^2)^2}$, $v = \frac{x^2 - y^2}{(x^2 + y^2)^2}$, $w = \frac{y}{x^2 + y^2}$ is irrotational. (5 marks)

QUESTION TWO (20 MARKS)

- a) Determine the analytical function $f(z) = u + iv$ if $u = x^3 - 3y^2 + 3x^2 - 3y^2 + 2x + 1$ (10 marks)
- b) If $w = \phi + i\psi$ represent the complex potential for an electric field and $\psi = x^2 - y^2 + \frac{x}{x^2 + y^2}$. Determine the function ϕ . (10 marks)

QUESTION THREE (20 MARKS)

- a) The ideal gas obeying Boyle's law $P = k\rho$ is rushing from a boiler through a conical pipe with radii a and b ; ($b > a$) at the two ends. If the gas enters a pipe from the narrow end with the velocity V and escapes through the other end with velocity U , show that
$$V = \frac{Ub^2}{a^2} \exp\left[\frac{V^2 - U^2}{2k}\right]$$
 (12 marks)
- b) Determine whether the flow with velocity components
$$u = \frac{-2xyz}{(x^2 + y^2)^2}, \quad v = \frac{(x^2 - y^2)z}{(x^2 + y^2)^2}, \quad w = \frac{y}{x^2 + y^2}$$
 is rotational or irrotational. (8 marks)

QUESTION FOUR (20 MARKS)

A two dimensional flow towards a normal boundary is found to be characterized by a normal component of the velocity that varies directly with distance from the boundary. Determine;

- a) The stream function (11 marks)
- b) The stream line (9 marks)

QUESTION FIVE (20 MARKS)

- a) Show that for a diabatic process $P\rho^{-r} = \text{constant}$; where $r = \frac{C_p}{C_v}$ (8 marks)
- b) Expressing the enthalpy S and internal energy E in terms of pressure P and volume V , show that;

i.)
$$\left(\frac{\partial S}{\partial P}\right)_V = \frac{C_v}{\alpha kT}$$
 (6 marks)

ii.) $\left(\frac{\partial S}{\partial V}\right)_P = \frac{C_P}{\alpha VT}$ (6 marks)

α – coefficient of volume expansion

k – Bulk modulus