MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING

BACHELOR OF EDUCATION (ARTS AND SCIENCE)

BACHELOR OF ECONOMICS AND STATISTICS

BACHELOR OF SCIENCE IN MATHEMATICS

BACHELOR OF ARTS

SMA 260 - PROBABILITY AND STATISTICS I

DATE: 19/01/2021 **TIME: 2.00-4.00 PM**

INSTRUCTIONS

1. *Answer Question* 1 *and any other* ***two*** *questions.*
2. *You must have a scientific calculator for this paper.*

QUESTION ONE

a) Outline **four** characteristics of the normal probability distribution. (2 marks)

b) A discrete random variable *x* has a probability function given by:

x	0	1	2	3	4	5
$f(x)$	$\frac{2}{20}$	$\frac{3}{20}$	$\frac{6}{20}$	$\frac{5}{20}$	$\frac{3}{20}$	$\frac{1}{20}$

(i) Show that the function $f(x)$ is a probability mass function (p.m.f.). (2 marks)

(ii) Determine the mean and the variance of the random variable x in the probability distribution. (6 marks)

(iii) Determine the probability that x is at least 3. (2 marks)

c) Two fair six faced dice are tossed. A random variable x is defined as the difference of the outcomes of the two dice.

(i) Derive the possibility space for the random variable x ; (4 marks)

(ii) Determine the distribution function of x expressed in tabular form; (4 marks)

d) The operational lifespan of a given brand of desktop computers has been found to be normally distributed with a mean of 4.8 years and a standard deviation of 1.6 years.

(i) Determine the proportion of the desktop computers that will have a lifespan of between 3.8 years and 6.6 years; (4 marks)

(ii) If these desktop computers have a warranty period of 2 years, determine the proportion of original sales which will require replacement through this warranty.(3 marks)

(iii) If the manufacturer of these desktop computers wants only 5% of the computers to be replaced through this warranty, determine the warranty period that should be set to achieve this. (3 marks)

QUESTION TWO

A continuous random variable x is given by the function

$$f(x) = \begin{cases} \frac{2}{9}(9x - x^2 - 18) \ ; & \text{for} \quad 3 \le x \le 6. \\ 0 & \text{elsewhere} \end{cases}$$

a) Show that the function $f(x)$ is a probability density function (p.d.f.). (3 marks)

b) Determine the following measures about the random variable x in the probability distribution:

(i) the mode of x ;

(ii) the mean of x ;

(iii) the variance of x . (13 marks)

c) Determine the following probability from the distribution:

$P(x \le 5)$. (4 marks)

QUESTION THREE

a) Outline **three** characteristics of the binomial probability distribution. (3 marks)

b) National Transport and safety Authority claims that only 60% of the drivers on Kenyan roads were trained in driving schools. A random sample of 12 drivers on the road was taken from Kenya. Determine the probability that among these drivers the following were trained in a driving school:

(i) exactly 5 drivers;

(ii) at least 6 drivers;

(iii) between 4 and 7 drivers inclusive. (10 marks)

c) It has been observed that 2 out of every 40 match sticks in a box coming out of a production process fail to light. Determine the probability that among 200 such match sticks randomly selected from the production process:

(i) exactly 8 will fail to light;

(ii) at least 3 will fail to light. (7 marks)

QUESTION FOUR

The marks scored in Mathematics by students who sat for KCSE examination in certain year has been found to be normally distributed with mean 54 marks and standard deviation 12 marks.

(a) (i) Suppose the pass-mark is set at 42 marks, determine the proportion of the students who will pass. (3 marks)

(ii) Determine the proportion of the students who will score a Credit grade if a Credit is assigned for marks between 60 and 74. (7 marks)

(b) (i) If the top 70% of the students are supposed to pass this examination, determine the mark which should be set as the pass-mark to achieve this. (3 marks)

(ii) Grades for results are awarded as follows:

- *Fail* to the bottom 25%,
- *Pass* to the next 30%,
- *Credit* to the next 20%,
- *Distinction* to the top 25%.

Determine the lower and upper limits of the range of the marks for the grades Pass and Credit. (7 *marks*)

QUESTION FIVE

(a) A continuous random variable x has an exponential probability distribution with probability density function (p.d.f.) given by:

$$f(x) = \lambda e^{-\lambda x} \quad \text{for} \quad 0 \le x \le \infty, \quad \lambda > 0.$$

Prove that the variance of x is given by: $var(x) = \frac{1}{\lambda^2}$ (10 *marks*)

(b) A continuous random variable x has a probability density function given by:

$$f(x) = \begin{cases} 2x \; ; & \text{for} \quad 0 \leq x \leq 1. \\ 0 \quad \text{elsewhere} & \end{cases}$$

(i) Determine the probability density function of a continuous random variable $y = 8x^3$ using the change of variable technique. (5 *marks*)

(ii) Hence determine the following:

I. the mean of y ;

II. the probability $P(Y > 4)$; (5 *marks*)