MACHAKOS UNIVERSITY COLLEGE

(A Constituent College of Kenyatta University)
University Examinations for 2015/2016 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST SEMESTER EXAMINATIONS FOR THE DEGREE IN

BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING

BACHELOR OF SCIENCE IN MATHEMATICS

SMA 230: VECTOR ANALYSIS 1

Date: 4/8/2016 **Time: 8:30 – 10:30 AM**

INSTRUCTIONS

Answer question ONE and any other TWO questions

QUESTION ONE (30 MARKS)

a) Given that $A = 5\hat{i} - 2\hat{j} + 3\hat{k},\ \vec{B} = 3\hat{i} + \hat{j} - 2\hat{k}\ \ and\ \ \vec{C} = \hat{i} - 3\hat{j} + 4\hat{k}$. Determine the following;

i. $\vec{A}\cdot\left(\vec{B}\times\vec{C}\right)$ (4 marks)

ii. $\vec{A}\times\left(\vec{B}\times\vec{C}\right)$ (5 marks)

iii. If $\vec{A} = t^2\,\hat{i} - t\,\hat{j} + (2t-1)\hat{k},\ \ and\ \ \vec{B} = (2t-3)\hat{i} + \hat{j} - t\,\hat{k}$, evaluate $\dfrac{d}{dt}\left[\vec{A}\times\dfrac{d\vec{B}}{dt}\right]$ at the point $t = 1$ (3 marks)

b) Calculate the equation of the plane determined by the points $P_1(2,-1,1)$, $P_2(3,2,-1)$ and $P_3(-1,3,2)$ (4 marks)

c) Evaluate the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1,-2,-1)$ in the direction $2\hat{i} - \hat{j} - 2\hat{k}$. (4 marks)

d) Calculate the work done in moving a particle once around a circle C in the $x-y$ *plane*, if the circle has center at the origin and radius 3 *units* and if the force field is given by

$\vec{F}=(2x-y+z)\hat{i}+(x+y-z)\hat{j}+(3x-2y+4z)\hat{k}$ (5 marks)

e) Evaluate $\iint(\nabla\times\vec{F})\cdot\hat{n}\,ds$, where $\vec{F}=(x^2+y-4)\hat{i}+3xy\hat{j}+(2xz+z^2)\hat{k}$ and S is the surface hemisphere $x^2+y^2+z^2=16$ above the $x-y$ *plane*. (5 marks)

QUESTION TWO (20 MARKS)

a) Evaluate the line integral $\oint xy\,dx+(2x-y)dy$ around the region bounded by the curve $y=x^2$ and $x=y^2$ by the use of Green's theorem. (5 marks)

b) Determine $\iint(\nabla\times\vec{F})\cdot\hat{n}\,ds$ over the entire surface of the region above the $x-y$ *plane* bounded by the cone $z^2=x^2+y^2$ and the plane $z=4$ if $F=4xyz\hat{i}+xyz^2\hat{j}+3z\hat{k}$ (5 marks)

c) Prove that the vector $\dfrac{\vec{r}}{r^2}$ is irrotational (5 marks)

d) Evaluate $\nabla^2(\ln r)$ (5 marks)

QUESTION THREE (20 MARKS)

a) Evaluate $\iiint\limits_V\left(\sqrt{x^2+y^2}\right)dxdydz$ where V is the region bounded by $z=x^2+y^2$ and $z=8-(x^2+y^2)$ (7 marks)

b) Consider the force field $\vec{F}=(2xy+z^3)\hat{i}+x^2\hat{j}+3xz^2k$;

   i) Show that the force field is conservative (4 marks)

   ii) Calculate the scalar potential (5 marks)

   iii) Determine work done in moving an object in the field from (1,-2, 1) to (3, 1, 4) (4 marks)

QUESTION FOUR (20 MARKS)

a) Prove that $\vec{r} = e^{-t}\left[c_1 \cos 2t + c_2 \sin 2t\right]$ satisfies the differential equation $\frac{d^2\vec{r}}{dt^2} + 2\frac{d\vec{r}}{dt} + 5\vec{r} = 0$ where c_1 *and* c_2 are constants. (7 marks)

b) Calculate $\hat{T}$, k *and* $\hat{N}$ for the curve $\vec{r} = -(3\sin 2t)\hat{i} - (3\cos t)\hat{j}$ (9 marks)

c) Given that $\vec{A} = t^2\hat{i} - t\,\hat{j} + (2t+1)\hat{k}$ and $\vec{B} = (2t-3)\hat{i} + \hat{j} - t\,\hat{k}$. Determine ;

$\frac{d}{dt}\left(\vec{A}\cdot\vec{B}\right)$ (4 marks)

QUESTION FIVE (20 MARKS)

a) Calculate the volume of the parallelepiped whose edges are represented by

$\vec{A} = 2\hat{i} - 3\,\hat{j} + 4\hat{k}$, $\vec{B} = \hat{i} + 2\,\hat{j} - \hat{k}$ *and* $\vec{C} = 3\hat{i} - \hat{j} + 2\hat{k}$ (5 marks)

b) Given that $\vec{A} = 3\hat{i} - 2\,\hat{j} + 4\hat{k}$ and $\vec{B} = 2\hat{i} - 3\,\hat{j} - 2\hat{k}$. Calculate the angle θ between the vectors $\vec{A}$ *and* $\vec{B}$ (5 marks)

c) Given that

$$\vec{a}' = \frac{\vec{b}\times\vec{c}}{\vec{a}\cdot\vec{b}\times\vec{c}}$$

$$\vec{b}' = \frac{\vec{c}\times\vec{a}}{\vec{a}\cdot\vec{b}\times\vec{c}}$$

$$\vec{c}' = \frac{\vec{a}\times\vec{b}}{\vec{a}\cdot\vec{b}\times\vec{c}}$$

show that if $\vec{a}\cdot\vec{b}\times\vec{c} = V$, then

$\vec{a}'\cdot\vec{b}'\times\vec{c}' = \frac{1}{V}$ (5 marks)

d) Consider the vectors;

$\hat{i} - 2\,\hat{j} + 4\hat{k}$, $2\hat{i} + 3\,\hat{j} + \hat{k}$, $-2\hat{i} + \hat{j} + \hat{k}$. Show that the three vectors are orthogonal.

(5 marks)