MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (PROGRAMMING AND STATISTICS)

BACHELOR OF SCIENCE (INFORMATION TECHNOLOGY)

BACHELOR OF EDUCATION (SPECIAL NEEDS)

BACHELOR OF ECONOMICS AND STATISTICS

BACHELOR OF SCIENCE (ACTUAL SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

BACHELOR OF ECONOMICS

SMA 231: DYNAMICS I

DATE: 9/12/2021 **TIME: 2.00-4.00 PM**

INSTRUCTION:
Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) A particle of mass 4 units move in such a way that it describes a space curve defined by;

$\vec{r} = (8t^4 - t^2)\hat{i} - 6t^2\hat{j} + (t^5 - 4)\hat{k}$. Calculate;

i. Velocity (3 marks)

ii. Momentum (3 marks)

iii. Acceleration (3 marks)

iv. Force field at any time t (3 marks)

b) A conservative force field defined by $\vec{F} = \left(y^2 z^3 - 6xz^2\right)\hat{i} + 2xyz^3\,\hat{j} + \left(3xy^2 z^2 - 6x^2 z\right)\hat{k}$ has a scalar function defined by $\varphi = 3x^2 z^2 - xy^2 z^3$. Calculate the work done by the force field in moving a particle from the point $A\ (-2, 1, 3)$ to the point $B\ (1, -2, -1)$ (3 marks)

c) Derive the equation $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$ for the simple harmonic oscillator (3 marks)

d) Show that the differential equation for a particle with damped harmonic motion is given by;

$$\ddot{x} + 2\gamma\dot{x} + w^2 x = 0$$ (4 marks)

e) prove that the torque acting on a particle equals to the time rate of change in angular momentum that is $\vec{\Lambda} = \frac{d\Omega}{dt}$ (4 marks)

f) Determine the instantaneous power applied to a particle of mass 5 units moving along a space curve whose position vector is given as a function of time t by;

$r = (2t^3 + t)\,\hat{i} + (3t^2 - t^2 + 8)\,\hat{j} - 12t^2\hat{k}$ (4 marks)

QUESTION TWO (20 MARKS)

A particle of mass $20kg$ moves with simple harmonic motion on the $x-axis$, initially its located at the distance of $4m$ away from the origin $(x = 0)$ and has a velocity of $15m/s$ and acceleration of $100m/s^2$ directed towards $(x = 0)$. Determine;

a) The position of the particle at any time t (10 marks)

b) Amplitude, period and frequency of the particle's vibration (3 marks)

c) The force on the particle when $t = \frac{\pi}{10}\sec onds$ (7 marks)

QUESTION THREE (20 MARKS)

a) A particle of mass m moves along a space curve defined by $\vec{r} = a\cos wt\ \hat{i} + b\sin wt\ \hat{j}$.

Determine;

i. The torque (5 marks)

ii. The momentum about the origin (5 marks)

b) The total work done in moving a particle along a curve from points P_1 to P_2 is given by $w = \int_c \vec{F}\cdot d\vec{r} = \frac{1}{2}m\left(\vec{v}_2^{\,2} - \vec{v}_1^{\,2}\right)$. Prove this theorem (10 marks)

QUESTION FOUR (20 MARKS)

A particle of mass m is projected with a velocity v_0 at an angle α with the horizontal. Assuming that the gravitational acceleration is constant and that the forces due to air resistance can be neglected. Determine;

a) The time of flight (10 marks)

b) The range on the horizontal plane of the particle (10 marks)

QUESTION FIVE (20MARKS)

a) A particle P of mass m slides without rolling down a frictionless inclined plane AB of angle α with the horizontal. If it starts from rest at the top A of the inclined plane. Calculate the particles;

   i. Acceleration (4 marks)

   ii. Velocity (4 marks)

   iii. Distance covered (4 marks)

b) An object slides on a surface of ice along the horizontal straight line OA. At a certain point in its path the speed is v_0 and the object comes to rest after travelling a distance x_0. Prove that the coefficient of friction is $\frac{v_0^2}{2gx_0}$ (8 marks)