



MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

SECOND YEAR SUPPLEMENTARY EXAMINATION FOR \\
BACHELOR OF EDUCATION

SMA 305: COMPLEX ANALYSIS I

DATE: 26/7/2019

TIME 11:00 – 1:00 PM

Instructions to the Candidate:

Attempt question *one (compulsory)* and any other *two questions*.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Define

- i. Complex number (1 mark)
- ii. Limit of a function (1 mark)
- iii. Entire function (1 mark)
- iv. Poles (1 mark)

b) Consider the function $f(z) = \frac{z - \sin z}{z^2}$

- i. State the singularity of $f(z)$ (1 mark)
- ii. What kind of singularity of $f(z)$ (2 marks)
- iii. Find the Laurent series of $f(z)$ (3 marks)
- iv. What region of convergence of $f(z)$ (1 mark)

c) Prove that $\int_0^{\infty} \frac{dx}{x^2+1} = \frac{\pi}{2}$ (8 marks)

d) Find all the 6th roots of unity (10 marks)

QUESTION TWO (20 MARKS)

- a) Let $f(z) = \frac{z^2 - 7z + 10}{(z-3)^2}$
- Determine the poles of $f(z)$ (2 marks)
 - Calculate the residue s of $f(z)$ at its poles (3 marks)
- b) Explain the nature of the transformation $w = z^2$ (5 marks)
- c) Evaluate $\int z dz$ from $z = 0$ to $z = 4 + 2i$ along the line $z = 2i$ to $z = 4 + 2i$ (10 marks)

QUESTION THREE (20 MARKS)

- a) Find the image of the point $(4,3)$ on z -plane under the transformation $w = 2z^2 + 3$ (4 marks)
- b) Let $f(z) = u + iv$ be an analytic function. If $u = x^3 - 3x^2y$, find $f(z)$ (4 marks)
- c) Simplify $\frac{(\cos 4\theta + i \sin 4\theta)(\cos 3\theta + i \sin 3\theta)}{(\cos \theta - i \sin \theta)(\cos 2\theta - i \sin 2\theta)}$ (5 marks)
- d) Expand $f(z) = \frac{3}{z^2(z-3)^2}$ in a Laurent series at $z = 3$ (7 marks)

QUESTION FOUR (20 MARKS)

- a) Find all the values of z such that $e^z = 2$ (4 marks)
- b) Prove that $f(z) = 2z^2$ is a continuous at $z = z_0$ (6 marks)
- c) Find the bilinear transformation which maps the points $z_1 = 2, z_2 = i, z_3 = -2$ into the points $w_1 = 1, w_2 = i, w_3 = -1$ respectively. (10 marks)

QUESTION FIVE (20 MARKS)

Evaluate the improper real integral $\int_0^\infty \frac{x^{p-1}}{1+x} dx$

Where p is a positive proper fraction. (20 marks)