

Machakos University
SECOND YEAR EXAMINATION FOR THE BACHELOR OF
EDUCATION (SCHOOL BASED)
SMA 260: PROBABILITY & STATISTICS 1

INSTRUCTION: Answer Question *ONE* which is compulsory and any other *TWO* Questions

Question One (30 marks)

- a) By citing examples distinguish the following statistical terms (4 marks)
- i) Discrete and continuous variables
 - ii) Cumulative distribution functions and Probability distribution functions
- b) If X is a hypergeometric random variable, show that $E(x) = \left(\frac{A}{N}\right)n$ (6 marks)
- c) Given $X \sim N(50, 10^2)$ determine $P(45 < x < 62)$ (4 marks)
- d) Suppose two percent of the output of a machine is defective, and a lot of 300 pieces is produced. Determine the probability that exactly four pieces will be defective. (5 marks)
- e) Highlight three properties of a binomial distribution (3 marks)
- f) Suppose X is a random variable with a density function defined as
- $$f(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \quad \lambda > 0 \\ 0 & , \quad \text{otherwise} \end{cases}$$
- Determine the mgf_x , $E(x)$ and $Var(x)$ (8 marks)

Question Two (20 marks)

- a) A survey interviewed 200 households on the family size (Children), the findings were as summarized below

Family size	0	1	2	3	4	5	6
Households	3	38	83	52	18	5	1

Using the information in the table

- i) Construct the probability distribution table
 - ii) Determine the expected value, variance and standard deviation of the distribution (10 marks)
- b) The *pdf* of a random variable X is given by

$$f(x) = \begin{cases} \frac{1}{6} & , -1 < x < 2 \\ 0 & otherwise \end{cases}$$

Calculate $E(x^2)$, $E(x^3)$ and μ_3 (10 marks)

Question Three (20 marks)

- Highlight four properties of a normal distribution (4 marks)
- The students' fees balance at a certain department is approximately normally distributed with an average balance of KShs 8000 and a standard deviation of KShs 3000. What is the probability that a student from the department randomly selected has a fees balance
 - Over KShs 12500;
 - Between KShs 6500 and KShs 9500. (8 marks)
- A pair of fair dice is tossed to obtain equiprobable space Ω consisting of 36 order pairs of numbers between 1 and 6. Determine the $f(x)$ and $F(x)$, represent $F(x)$ in a graph (8 marks)

Question Four (20 marks)

- A test consists of 6 questions, and to pass the test a student has to answer at least 4 questions correctly. Each question has three possible answers, of which only one is correct. If a student guesses on each question, what is the probability that the student
 - Got no answer correct
 - Passed the examination (6 marks)
- If the probability that an individual suffers a bad reaction from a certain injection is 0.001, determine the probability that out of 2000 individuals, more than 2 individuals will suffer a bad reaction. (8 marks)
- Given that $F(x)$ is the CDF of a poisson distribution with parameter λ and that $F(2) = 2F(1)$. Determine the value of λ (6 marks)

Question Five (20 marks)

- Ships arrive in a harbour at a mean rate of two per hour. Suppose that this situation can be described by a Poisson distribution. Find the probabilities for a 30-minute period that
 - No ships arrive;
 - Three ships arrive. (6 marks)
- The diameter of an electric cable (x) is assumed to be continuous random variable with such that $f(x) = 6x(1-x)$ for $0 < x < 1$ otherwise it is zero.
 - Confirm that $f(x)$ is a pdf
 - Determine a number b such that $p(x < b) = p(x > b)$ (8 marks)

c) Given that X is a random variable with *pdf*

$$f(x) = \begin{cases} \frac{1}{2\sqrt{x}} & 1 < x < 4 \\ 0 & \text{elsewhere} \end{cases}$$

Determine

- i. The quartile deviation of the random distribution
 - ii. The 30th percentile of the random distribution
- (6 marks)