MACHAKOS UNIVERSITY

University Examinations 2022/2023 Academic year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

SECOND YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (APPLIED PHYSICS AND TECHNOLOGY)

SPT 201: MATHEMATICS FOR PHYSICS II

DATE: 11/4/2023 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS:

Answer question **ONE** which is compulsory and any other **TWO**

SECTION A

QUESTION ONE (30 MARKS)

(a) (i) Distinguish between dot and cross product of a vector (2 marks)
(ii) Give examples of physical situations where each product in a(i) above may be used (2 marks)

(b) Vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ are defined as $\boldsymbol{A} = 4\boldsymbol{i} + 3\boldsymbol{j} + 12\boldsymbol{k}$ and $\boldsymbol{B} = 8\boldsymbol{i} - 6\boldsymbol{j}$. Determine
(i) Unit vectors $\widehat{\boldsymbol{A}}$ and $\widehat{\boldsymbol{B}}$ (4 marks)
(ii) Angle between $\boldsymbol{A}$ and $\boldsymbol{B}$ using the cross product (3 marks)

(c) For the vectors in (b), show that $\boldsymbol{A} \times \boldsymbol{B} = -\boldsymbol{B} \times \boldsymbol{A}$ (4 marks)

(d) A force is given by $\boldsymbol{F} = 2\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$. Determine
(i) The work done by the force in the direction of $\boldsymbol{r} = 3\boldsymbol{i} + 4\boldsymbol{j} - 2\boldsymbol{k}$. (2 marks)
(ii) Angle between $\boldsymbol{F}$ and $\boldsymbol{r}$ using dot product (3 marks)

(e) Show that the points A(1, 2, 3), B(3, 8, 1) and C(7, 20, -3) are collinear (4 marks)

(f) Show that $\boldsymbol{A} = \boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k}$, $\boldsymbol{B} = 2\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{C} = 3\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$ are co-planer vectors (4 marks)

(g) Show that $\boldsymbol{F} = (2xy + z^3)\boldsymbol{i} + x^2\boldsymbol{j} + 3xz^2\boldsymbol{k}$ is a conservative force vector (3 marks)

SECTION B

ATTEMPT ANY TWO QUESTIONS

QUESTION TWO (20 MARKS)

(a) A particle moves in space such that at any time t, its position, **r** is given by $x = 2t + 3$, $y = t^2 + 3t$ and $z = t^3 + 2t^2$. At the time $t = 1$ second, determine the following parameters along the path given by $\boldsymbol{P} = 2\boldsymbol{i} + 3\boldsymbol{j} + 4\boldsymbol{k}$

(i) Velocity (4 marks)

(ii) Acceleration (4 marks)

(b) The path described by a particle is given by $x = 2u$, $y = u^2 + 3$ and $z = 2u^2 + 5$. Determine the unit tangent vector at the point (2,4,7) (4 marks)

(c) A charged particle is moving at a velocity given by $\boldsymbol{V} = 2u\boldsymbol{i} - 3u\boldsymbol{j} + (u - 2)\boldsymbol{k}$ within a magnetic field $\boldsymbol{B} = 3u\boldsymbol{i} + u^2\boldsymbol{j} + (u + 2)\boldsymbol{k}$. Evaluate

(i) the force $\boldsymbol{F}$ if $\boldsymbol{F} = \boldsymbol{B} \times \boldsymbol{V}$**,** at $u = 1$ (4 marks)

(ii) Hence evaluate $\int_0^2 (\boldsymbol{B} \times \boldsymbol{V})\boldsymbol{du}$ (4 marks)

QUESTION THREE (20 MARKS)

The vector fields $\boldsymbol{A} = x^2y\boldsymbol{i} + (xy + yz)\boldsymbol{j} + xz^2\boldsymbol{k}$ and $\boldsymbol{B} = yz\boldsymbol{i} - 3xz\boldsymbol{j} + 2xy\boldsymbol{k}$ as well as a scalar field $\emptyset = 3x^2y + xyz - 4y^2z^2 - 3$ exist within a certain region, determine the following parameters

(a) Grad $\emptyset$, (3 marks)

(b) Div $\boldsymbol{A}$ (3 marks)

(c) Grad $(\boldsymbol{A}.\boldsymbol{B})$ (4 marks)

(d) Curl $\boldsymbol{A}$ (4 marks)

(e) Curl Div $\boldsymbol{B}$ (6 marks)

QUESTION FOUR (20 MARKS)

(a) Show that the

(i) Equation $x^2y" + xy' + (x^2 - p^2)y = 0$ is a singular point of the Bessel function (2 marks)

(ii) Bessel function of order zero is given by $J_0(x) = 1 - \frac{x^2}{2^2} + \frac{x^4}{2^2\,4^2} - \frac{x^6}{2^2\,4^2\,6^2} + \cdots$ (5 marks)

(b) (i) State the Greens' theorem (2 marks)

(ii) Use the Greens' theorem to evaluate the closed path integral $\oint\{(x^2 + y^2)dx + (x + 2y)dy\}$ taken round the boundary curve given by *y* = 0, $0 \le x \le 2$; $x^2 + y^2 = 4$, $0 \le x \le 2$ and *x* = 0, $0 \le y \le 2$ (6 marks)

(c) Find the Laplace transform for the function, *f(t) = sin at* (5 marks)

QUESTION FIVE (20 MARKS)

(a) Distinguish Hermite and Laguerre polynomials using their characteristic equations as well as solutions (4 marks)

(b) Determine the Legendre polynomial $p_2(x)$ (5 marks)

(c) Solve the differential equation, $\frac{dy}{dx} = y$

(i) Using separation of variables method (3 marks)

(ii) As a power series, in ascending powers of x (6 marks)

(iii) Hence show that the two solutions in c(i) and (ii) are equivalent (2 marks)