



MACHAKOS UNIVERSITY

University Examinations for 2020/2021 Academic Year

SCHOOL OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF BUILDING AND CIVIL ENGINEERING

THIRD YEAR SPECIAL/ SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN CIVIL ENGINEERING

ECV 312: THEORY OF STRUCTURES IV

DATE: 26/3/2021

TIME: 11.00-1.00 PM

INSTRUCTIONS

Answer Question One and Any Other Two Questions

QUESTION ONE (20 MARKS)

Determine the reactions and the force in each member of the truss shown in the Figure 1 below by finite element method. $EA = \text{constant}$

QUESTION TWO (20 MARKS)

Determine the reactions and the member end forces for the three-span continuous beam shown in the Figure 2 below using finite element method. $E = 200 \text{ GPa}$, $I = 216(10^6) \text{ mm}^4$

QUESTION THREE (20 MARKS)

Determine the reactions and the member forces the two-member frame shown in the Figure 3 below. Take $I = 243(10^6) \text{ mm}^4$, $A = 6000 \text{ mm}^2$ and $E = 200 \text{ GPa}$ for both members.

QUESTION FOUR (20 MARKS)

Determine the reactions and the member end forces of the frame shown in the Figure 4 below. Take $I = 250(10^6) \text{ mm}^4$, $A = 900 \text{ mm}^2$ and $E = 200 \text{ GPa}$ for each member.

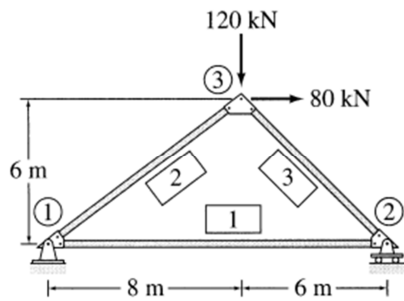


Figure 1

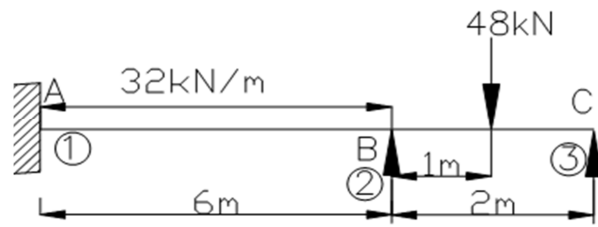


Figure 2

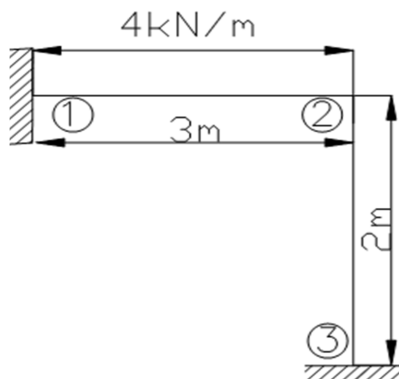


Figure 3

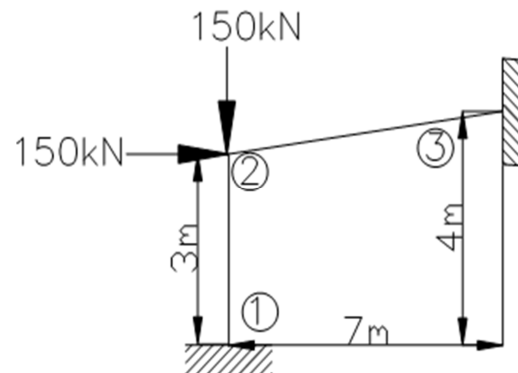


Figure 4

HINTS

$$\begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

$$\lambda_x = \cos \theta_x$$

$$\lambda_y = \cos \theta_y$$

$$\begin{pmatrix}
\left(\frac{AE}{L} \lambda_x^2 + \frac{12EI}{L^3} \lambda_y^2\right) & \left(\frac{AE}{L} - \frac{12EI}{L^3}\right) \lambda_x \lambda_y & -\frac{6EI}{L^2} \lambda_y & -\left(\frac{AE}{L} \lambda_x \lambda_{yx} + \frac{12EI}{L^3} \lambda_y \lambda_{yy}\right) & -\left(\frac{AE}{L} \lambda_x \lambda_{yy} - \frac{12EI}{L^3} \lambda_y \lambda_{yx}\right) & -\frac{6EI}{L^2} \lambda_y \\
\left(\frac{AE}{L} - \frac{12EI}{L^3}\right) \lambda_x \lambda_y & \left(\frac{AE}{L} \lambda_y^2 + \frac{12EI}{L^3} \lambda_x^2\right) & \frac{6EI}{L^2} \lambda_x & -\left(\frac{AE}{L} \lambda_y \lambda_{yx} - \frac{12EI}{L^3} \lambda_x \lambda_{yy}\right) & -\left(\frac{AE}{L} \lambda_y \lambda_{yy} + \frac{12EI}{L^3} \lambda_x \lambda_{yx}\right) & \frac{6EI}{L^2} \lambda_x \\
-\frac{6EI}{L^2} \lambda_y & \frac{6EI}{L^2} \lambda_x & \frac{4EI}{L} & \frac{6EI}{L^2} \lambda_{yy} & -\frac{6EI}{L^2} \lambda_{yx} & \frac{2EI}{L} \\
-\left(\frac{AE}{L} \lambda_x \lambda_{yx} + \frac{12EI}{L^3} \lambda_y \lambda_{yy}\right) & -\left(\frac{AE}{L} \lambda_y \lambda_{yx} - \frac{12EI}{L^3} \lambda_x \lambda_{yy}\right) & \frac{6EI}{L^2} \lambda_{yy} & \left(\frac{AE}{L} \lambda_{yx}^2 + \frac{12EI}{L^3} \lambda_{yy}^2\right) & \left(\frac{AE}{L} - \frac{12EI}{L^3}\right) \lambda_{yx} \lambda_{yy} & \frac{6EI}{L^2} \lambda_{yy} \\
-\left(\frac{AE}{L} \lambda_x \lambda_{yy} - \frac{12EI}{L^3} \lambda_y \lambda_{yx}\right) & -\left(\frac{AE}{L} \lambda_y \lambda_{yy} + \frac{12EI}{L^3} \lambda_x \lambda_{yx}\right) & -\frac{6EI}{L^2} \lambda_{yx} & \left(\frac{AE}{L} - \frac{12EI}{L^3}\right) \lambda_{yx} \lambda_{yy} & \left(\frac{AE}{L} \lambda_{yy}^2 + \frac{12EI}{L^3} \lambda_{yx}^2\right) & -\frac{6EI}{L^2} \lambda_{yx} \\
-\frac{6EI}{L^2} \lambda_y & \frac{6EI}{L^2} \lambda_x & \frac{2EI}{L} & \frac{6EI}{L^2} \lambda_{yy} & -\frac{6EI}{L^2} \lambda_{yx} & \frac{4EI}{L}
\end{pmatrix}$$

FIXED END FORCES

At A

$$M_A = -\frac{Wab^2}{L^2}$$

$$R_A = -\frac{Wb}{L^3}(l^2 + ab - a^2)$$

At B

$$M_B = -\frac{Wa^2b}{L^2}R_A = -\frac{Wa}{L^3}(l^2 + ab - b^2)$$

