MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN MATHEMATICS

SMA 360: PROBABILITY AND STATISTICS IV

DATE: 21/1/2021 **TIME: 2.00-4.00 PM**

INSTRUCTIONS TO CANDIDATES

Answer question one and any other two questions

QUESTION ONE

a) Define the following terms giving illustrations;

i. A random vector (2 marks)

ii. A random matrix (2 marks)

b) Suppose $\underline{X}=(x_1, x_2, \ldots, x_p)^T$ and that $\underline{a}$ is a PX1 vector of constants. Show that $Var(\underline{a}^T\underline{X})= \underline{a}^T\sum \underline{a}$ where $\sum$ is a variance covariance matrix. (4 marks)

c) Suppose $\sum=\begin{bmatrix}4 & 1 & 2\\ 1 & 9 & -3\\ 2 & -3 & 25\end{bmatrix}$ Determine the correlation matrix, $\boldsymbol{\rho}$. (4 marks)

d) Suppose $X\sim N(\mu, \sigma)$. Find the characteristic function of $Z={(x-\mu)}/{\sigma}$. (5 marks)

e) Let $\underline{X}\sim N_p(\underline{\mu},\sum)$. Suppose $\underline{Y}=A\underline{X}+\underline{b}$ where A is a qxp matrix of constants and $\underline{b}$ is a non zero qX1 vector of constants. Determine the distribution of $\underline{Y}$ by use of characteristic function. (5 marks)

f) Compute the mean vector from the data matrix $\mathbf{X}=\begin{bmatrix} 4 & 1 \\ -1 & 3 \\ 3 & 5 \end{bmatrix}$ (4 marks)

g) Show that $\Sigma=\mathbf{E}(\underline{XX}^{\mathbf{T}})-\underline{\mu\mu}^{\mathbf{T}}$ (4 marks)

QUESTION TWO

a) Given two random variables X and Y with a joint probability function $P_{xy}(x,y)$ represented below;

Y

		0	1
	-1	0.24	0.06
X	0	0.16	0.14
	1	0.40	0.0

Find the variance covariance matrix, Σ. (10 marks)

b) Suppose $\underline{X}=(X_1, X_2, X_3)^{\mathbf{T}}$ with $\underline{\mu}=(2,5,3)^{\mathbf{T}}$ and $\Sigma=\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 4 \end{bmatrix}$. Find;

i. Distribution of (x_1, x_2).

ii. Distribution of $(x_1, x_3/x_2=2)$

iii. $R_{2.13}$ (10 marks)

QUESTION THREE

a) Given that $f(x_1,x_2)$ is a bivariate normal density function. Show that $f(x_1/x_2)$ is $N\{\mu_1+\frac{\sigma_{12}}{\sigma_{22}}(x_2-\mu_2), \sigma_{11}(1-\rho_{12}^2)\}$. (10 marks)

b) Given that $f(x_1,x_2)=\boldsymbol{\alpha} e^{-\frac{1}{2}X}$ where α is some constant and $X=X_1^2+2X_2^2-X_1X_2-3X_1-2X_2$. Identify $\underline{\mu}$, Σ and hence express $\boldsymbol{\alpha}$ in terms of $\boldsymbol{\pi}$.

(10 marks)

QUESTION FOUR

a) Let X_1, X_2, X_3, and X_4 be independent and identically distributed 3 X 1 random vectors with $\underline{\mu}=\begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$ and $\Sigma=\begin{bmatrix} 3 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$**.** Consider two linear combinations of the random vectors $\frac{1}{2}X_1+\frac{1}{2}X_2+\frac{1}{2}X_3+\frac{1}{2}X_4$ and $X_1+X_2+X_3-3X_4$. Find;

i. The mean vector for each linear combination of vectors

ii. The covariance matrix for each linear combination of vectors and

iii. The covariance between the two linear combination of vectors (10 marks)

b) Given that **D**=diagonal(σ_1, σ_2, …, σ_p). Show that Σ**= D ρ D** where Σ is a variance covariance matrix and **ρ** is a correlation matrix. (4 marks)

c) Suppose $\underline{X} \sim N_3(\underline{\mu}, \Sigma)$ and given that $Y_1=X_1-X_2$ and $Y_2=X_1-X_3$. Find

i. $E(\underline{Y})$ (2 marks)

ii. $Var(\underline{Y})$ (4 marks)

QUESTION FIVE

a) Suppose $\underline{X}\sim N_p(\underline{\mu},\Sigma)$ and $Q=(\underline{x}-\underline{\mu})^T\Sigma^{-1}(\underline{x}-\underline{\mu})$. Show that $Q\sim \chi^2_p$ (5 marks)

b) Determine the **E**(**Q**) given that **Q**=$X_1^2+2X_2^2+3X_3^2+3X_1X_2+4X_1X_3$, $\underline{\mu}=\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\Sigma=\begin{bmatrix} 4 & 3 & 2 \\ 3 & 2 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ (5 marks)

c) Suppose $Y=X_1+2X_2+X_3$ and $Z= X_1-X_2+3X_3$. Determine the coefficient of correlation between Y and Z given that $Var(\underline{X}) = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 4 \end{bmatrix}$. (5 marks)

d) Let $\underline{Y}=(Y_1, Y_2, Y_3)$ having $\Sigma=\begin{bmatrix} 5 & 2 & 3 \\ 2 & 3 & 0 \\ 3 & 0 & 2 \end{bmatrix}$**.** Suppose that $X=Y_1-2Y_2+Y_3$. Compute Var(X).

(5 marks)