MACHAKOS UNIVERSITY

University Examinations 2023/2024

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR THE BACHELOR OF SCIENCE IN APPLIED PHYSICS AND TECHNOLOGY (APT) AND BACHELOR OF EDUCATION (SCIENCE), AND BACHELOR OF EDUCATION (SPECIAL NEEDS).

SPH 402: QUANTUM MECHANICS II

DATE: December 2023 **TIME: 2 Hours**

INSTRUCTIONS:

- The paper consists of **two** sections.
- Section **A** is **compulsory** (30 marks).
- Answer any **two** questions from section **B** (each 20 marks).

The following information may be useful:

Electron mass $= 9.11 \times 10^{-31}\mathrm{kg}$

Speed of light, $c = 3.0 \times 10^{8}\mathrm{m/s}$

Electron charge, $e = 1.6 \times 10^{-19}\mathrm{C}$

Laplacian operator in spherical coordinates

$$\nabla^2 = \frac{1}{r^2\sin\theta}\left[\sin\theta\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin\theta}\frac{\partial^2}{\partial\phi^2}\right]$$

Volume element $dv = r^2\sin\theta\, dr\, d\theta\, d\phi$

SECTION A

QUESTION ONE (30 MARKS)

a) State **Four** major properties of an atom that are learnt by using the Eigen functions (2 marks)

b) Give the expressions that relate the z-component of the orbital angular momentum (L_z) and the square of the orbital angular momentum (L^2) to the quantum numbers m_l and l respectively. Hence sketch the vector model of the orbital angular momentum for $l = 2$. (5 marks)

c) Determine the reduced mass of the hydrogen atom that consists of an electron $(mass\ \boldsymbol{m_e})$ and a proton $(mass\ \boldsymbol{m_p})$, both moving around a common centre (6 marks)

d) Write Schrödinger equation for a Hydrogen atom and explain all symbols used (3 marks)

e) Determine the magnitude of orbital angular momentum for an electron whose orbital angular momentum quantum number is 2 (2 marks)

f) State the meaning of the term '***degenerate***' as applicable to eigenfunctions. Hence, enumerate the degree of degeneracy for the eigenvalue E_2. (4 marks)

(g) Define the term '***Transition rate'***. The transition of hydrogen atom from its first exited state to its ground state is $R \equiv 10^8 s^{-1}$. Briefly explain the meaning of this statement (4 marks)

h) Explain the formation of excited states, giving the example of Beryllium. Simple illustrations of the orbital occupation for both the ground and excited states are necessary (2 marks)

i) Determine the number of electrons that can occupy the p-shell of an atom (2 marks)

SECTION B

QUESTION TWO (20 marks)

a) i) By using the method of separation of variables, reduce the Schrödinger equation into three ordinary differential equations (use the spherical coordinate system) (6 marks)

ii) Discuss the significance of each of the separation constants used and hence write the most general solution of the Schrödinger equation in terms of the separation constants (5 marks)

c) State the selection rules that must be satisfied by quantum numbers of the energy levels during the transitions corresponding to the observed discrete frequency photons. Also give expression for the frequency of the photons observed during the transitions (3 marks)

d) Enumerate possible values of the magnetic quantum number, μ for each of the Sub-shells in the N-shell of an atom, stating the sub-shells in your enumeration. Use the weaker form of Pauli Exclusion Principle to display the electronic configuration of Arsenic (atomic number = 33) in its ground state. (6 marks)

QUESTION THREE (20 marks)

a) Given that the wavefunction of a 1s electron is

$$\psi = \frac{e^{-r/a_0}}{\sqrt{\pi}{a_0}^{3/2}}$$

Where a_0 is the Bohr radius, verify that the average value of $\frac{1}{r}$ for a 1s electron is exactly $\frac{1}{a_0}$ (5 marks)

b) Using the appropriate selection rules, show how the transition $l = 2$ to $l = 1$ results to the Zeeman triplets in the presence of a magnetic field (4 marks)

c) Briefly describe the following as applied in the calculation of total angular momentum:

i) LS coupling

ii) jj coupling (4 marks)

d) The magnetic moment μ_j of an atom in which LS coupling holds has the magnitude

$$\mu_j = \sqrt{J(J+1)}g_j\mu_B$$

Where μ_B is the Bohr magneton

Show that the Lande' g-factor is given by

$$g_j = 1 + \frac{J(J+1) - L(L+1) + S(S+1)}{2J(J+1)}$$

(7 marks)

QUESTION FOUR (20 marks)

a) i) The orbital angular momentum operator $\underline{L}$ has three components L_x, L_y and L_z in the Cartesian coordinates system. Determine the nature of L_x, L_y and L_z in operator notation if the Schrödinger representation is used. (5 marks)

ii) Given that the eigenvalues of the operator L_z acting on the wavefunction $\psi_m(\phi)$ are $m\hbar$, determine the allowed value of m. (4 marks)

b) An atom drops from an energy level E_m to a lower energy E_n. Show that the frequency of the energy radiated is

$$\nu = \frac{E_m - E_n}{h}$$

where h is the Planck's constant (7 marks)

c) A sample of a certain element is placed in a 0.03T magnetic field and suitably excited. How far apart are the Zeeman components of the 450 nm spectral line of this element? (4 marks)

QUESTION FIVE (20 marks)

a) The orbital angular momentum operator

$$L^2 =- \hbar^2 \left[\frac{1}{sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$$

Can operate on the spherical harmonics $r_{lm}(\theta, \phi)$

i) By setting up the appropriate eigenvalue equation, show that the only physically acceptable solution when the magnetic quantum number $m = 0$ is of the form of Legendre's differential equation (5 marks)

ii) Use the Power Series method to solve the resulting Legendre equation and discuss the solutions. (6 marks)

b) Consider an electron orbiting a circular path of radius r at a frequency v

i) Determine its magnetic moment (3 marks)

ii) Hence or otherwise determine its magnetic potential energy if it is placed in a magnetic field B. Identify the quantity known as the Bohr magneton (6 marks)