MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SMA 433: PARTIAL DIFFERENTIAL EQUATIONS II

Time 2hours

Attempt question ***one (compulsory)*** and any other ***two questions***.

Question one (30marks)

a.) Define a second order linear partial differential equation (4marks)

b.) State the short form a second order partial differential equation (3marks)

c.) State the three classes of the second order partial differential equation (4marks)

d.) Consider the equation $2u_{xx} + 3u_{xy} + u_{yy} = 0$.

i.) Classify the equation (4marks)

ii.) Determine its characteristic equation (5marks)

iii.) Transform the equation to canonical form (7marks)

iv.) Solve the equation (7marks)

Question Two (20marks)

Prove the general equation $A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + D\frac{\partial u}{\partial x} + E\frac{\partial u}{\partial y} + Fu = 0$

Where A, B, C, D, E, F are real constants such that $A \neq 0$.

Question Three (20marks)

Solve the wave equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$

Given that $u(x,0) = f(x)$ and $u_t(x,0) = g(x)$.

Question four (20marks)

Determine the characteristic of the Tricomi equation $y u_{xx} + u_{yy} = 0$ in the lower-half plane $y < 0$. Put this equation into canonical form in the upper half- plane $y > 0$.

Question five (20marks)

a.) An infinitely long string having one end at $x = 0$ is initially at rest on the $x - axis$. At $t = 0$;the end $x = 0$ begins to move along the $u - axis$ in a manner described by $u(0,t) = a\cos\sigma t$. Determine the displacement $u(x,t)$ of the string at any point at any time. (10marks)

b.) Using Mong'es method obtain the integral of $q^2 r - 2pqs + p^2 t = 0$ in the form $y + xf(z) = F(z)$ (10marks)