MACHAKOS UNIVERSITY

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR, FIRST SEMESTER EXAMINATIONS

FOR DIPLOMA IN EDUCATION

SMA 0205: VECTOR ANALYSIS

Date Time

Instructions

Attempt question **one** and any other **two** questions.

Show all your working.

QUESTION ONE 30MKS

a) Define a i) Scalar product (2m)

ii) Vector product (2m)

b) Given that **P** = 5**i** – 2**j** + **k** and **Q** = 2**i** + 5**j** – 6**k** evaluate

i) **P. Q** (3m)

ii) **P** x **Q** (4m)

c) If **A** = (u+3) **i** – $(2u+u^2)$ **j** + $2u^3$**k** determine

i) $\frac{d\boldsymbol{A}}{du}$ ii) $\frac{d^2\boldsymbol{A}}{du^2}$ (4m)

d) If F = 2uv**i** + (v^2-5u) **j** – $(2u+v^2)$ **k** determine

i) $\frac{\partial \boldsymbol{F}}{\partial u}$ ii) $\frac{\partial^2 \boldsymbol{F}}{\partial u^2}$ iii) $\frac{\partial^2 \boldsymbol{F}}{\partial u \partial v}$ (6m)

e) i) Given $\emptyset = 2x^2y^3z$. Find grad $\emptyset$ (3m)

ii) Show that curl (-y**i** + x**j**) is a constant vector. (6m)

QUESTION TWO 20MKS

a) Given **A** = x^2y^3**i** – $2xyz^2$**j** + x^2z**k**

B = xy^2z**i** + $3yz^2$**j** – xyz^2**k** and that

$\emptyset = xy^2z^3 - 3xy^2 + xyz^2$

Determine at the point (1, -2,1)

i) $\nabla\emptyset$

ii) $\nabla . \mathbf{A}$

iii) $\nabla \times \mathbf{B}$ (10m)

b) If **OA** = 2**i** + 3**j** – **k** and **OB** = **i** – 2**j** + 5**k** **determine**

i) the value of **OA. OB** **(2mks)**

ii) the product **OA** x **OB** in terms of unit vectors (2mks)

iii) the cosine of the angle between **OA** and **OB** (6mks)

QUESTION THREE 20MKS

a) If **A** = 2**i** – 3**j** + **k, B** = **i** +4**j** – 2**k, and C** = 3**i** + **j** + 2**k**,

Determine

i)**A. B** **(2mks)**

ii) angle between **A** and **B** **(3mks)**

iii) **B** x **C** **(3mks)**

b) If A = x^2y**i** + (xy +yz) **j** + xz^2**k**

B = yz**i** – 3xz**j** + 2xy**k** and

$\emptyset = 3x^2y + xyz - 4y^2z^2 - 3$ determine at the point (1,2,1)

i) grad $\emptyset$ (3mks)
ii) div grad $\emptyset$ (3mks)
iii) grad div **A** **(3mks)**
iv) div curl **B** (3mks)

QUESTION FOUR 20MKS

a) Given that u = $e^x[\sin(y+z) - \ y cos(y+z)]$

Show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} + \ u = 2e^x \sin(y+z)$ (10mks)

b) Find the directional derivatives of the function $\emptyset = x^2z + 2xy^2 + yz^2$ at the point (1,2, -1) in the direction of the vector A = 2**i** + 3**j** – 4**k** (10mks)

QUESTION FIVE 20MKS

A surface consists of five sections formed by the planes x = 0, x = 1, y = 0, y = 3,

$Z = 2$ in the first octant. If the vector field $F = y\mathbf{i} + z^2\mathbf{j} + xy\mathbf{k}$ exists over the surface and around its boundary verify Stokes theorem.